OPAQUENESS AND LIQUIDITY IN OVER-THE-COUNTER MARKETS

Fernando Lopes¹ Angelo Mendes² Gabriel Toledo³

¹Wisconsin

²Minnesota

³NYU

March 2024

MOTIVATION

- OTC asset markets can be very opaque
 - Mortgage backed securities; key for 2008 Crisis (Gorton and Ordonez (2014))
- Parts might trade under double-sided uncertainty
- Acquiring the asset does not translate to knowing its quality

THIS PAPER

- OTC model à la Duffie, Gârleanu, and Pedersen (2005), with quality heterogeneity over the assets;
- *Ex-post* uninformed asset holders
- Uncertain sellers; affect the price of current trade
- Endogenous belief deterioration; affect prices of future trades

THIS PAPER

How are the dynamics of an OTC market when trade occurs under double-sided uncertainty?

- What kind of equilibria exist here?
- How do belief dynamics shape and are shaped by trade?
- This presentation:
 - i. Present general setting and mechanics
 - ii. Specialize to specific support, show SS results
 - iii. Compare with ex-post informed asset holders

MAIN CONTRIBUTIONS

- Gorton and Ordonez (2014): No problem when information is not being produced.
 - Our model: Markets can get *stuck*. Key is *re-trade*
- Chiu and Koeppl (2016): Multiple equilibria because of *lemons*;
 - Our model: Strategic complementarities generate multiplicity
- Hellwig and Zhang (2012): Informed sellers, no role for beliefs
 - Our model: Beliefs dynamics shape equilibrium distribution of agents
- Choi (2018): Learning rates. Learning-by-holding attains max welfare
 - Our model: Explicit role of belief deterioration and re-trade

SETUP

- $t \in \mathbb{R}_+$ discounted at r > 0
- S assets mature with rate δ and leave the model
 - Good assets pay ug upon maturity
 - Bad assets pay $0 < u_b < u_g$
 - Fraction $ar{q}\in$ (0, 1) of good assets
- 1 + S agents can be *holders*, *sellers*, or *buyers*
- Holders are unshocked investors with the asset
- Sellers when liquidity shock arrives at κ
 - Flow of -x and $\delta u_b x > 0$
- Both indexed by $q \in [0,1]$ private belief of holding a good asset



- Buyers don't own the asset, meetings arrive at $\boldsymbol{\lambda}$
- Make TIOLI offers
 - Uninformed holder with updated belief
- Matured agents leave the model and are replaced by *holders* with replenished beliefs according to α(q)
 - Specialize two-belief replenish, rates (α_0, α_1) at (q_0, q_1)

SETUP

- Distribution of agents $\mu_t = \{(\mu_{ht}(q), \mu_{st}(q))_{q \in [0,1]}, \mu_{bt}\}$
- Resources constraints:

$$\int_{0}^{1} \mu_{st}(q) dq + \int_{0}^{1} \mu_{ht}(q) dq = S$$
 (1)

and the good assets feasibility

$$\int_{0}^{1} q \mu_{st}(q) dq + \int_{0}^{1} q \mu_{ht}(q) dq = \bar{q} S$$
 (2)

BUYERS' PROBLEM

- Reservation prices for each seller $q: P_t(q) = V_{st}(q) V_{bt}$
- Guess $V_{st}(q)$ is increasing, so are $P_t(q)$
- Gains from trade under $P_t(q)$

$$\underbrace{\frac{\int_{0}^{q} \mu_{st}(\tilde{q}) d\tilde{q}}{\int_{0}^{1} \mu_{st}(\tilde{q}) d\tilde{q}}}_{\mathcal{M}(q)} \times \left[V_{ht}(\pi_{t}(q)) - V_{st}(q) \right]$$

• With $\pi_t(q) = \frac{\int_0^q \tilde{q} \mu_{st}(\tilde{q}) d\tilde{q}}{\int_0^q \mu_{st}(\tilde{q}) d\tilde{q}} \le q$, posterior belief after trade under $P_t(q)$

• Probability of offering each $P_t(q)$: $\{\hat{\psi}_t(q)\}_{q \in [0,1]}$

VALUE FUNCTIONS

- Let $u(q) = qu_g + (1 q)u_b$
- Holders

$$rV_{ht}(q) = \kappa \left(V_{st}(q) - V_{ht}(q) \right) + \delta \left(u(q) - V_{ht}(q) \right) + \dot{V}_{ht}(q)$$

Sellers

$$rV_{st}(q) = \delta \left(u(q) - V_{st}(q) \right) - x + \lambda \left\{ \int_{q}^{1} \psi_{t}(\tilde{q}) \left(\underbrace{V_{st}(\tilde{q})}_{V_{b} + P_{t}(\tilde{q})} - V_{st}(q) \right) d\tilde{q} \right\} + \dot{V}_{st}(q)$$

Today: look for SS pure strategy equilibria

DYNAMICS



▶ LOM ▶ Equilibrium

TWO-BELIEF REPLENISH

- Now agents enter at a pair $q_0, q_1 \in [0, 1]$ at rates α_0, α_1
- Need $\alpha_0 + \alpha_1 = \delta$ and $\frac{\alpha_0 q_0 + \alpha_1 q_1}{\delta} = \bar{q}$
- We construct SS equilibrium with high price $(P(q_1))$ and low price $(P(q_0))$
 - Is there any other? No!

HIGH PRICE EQUILIBRIUM

Sellers



HIGH-PRICE EQUILIBRIUM

•
$$\mu_h(q) = \mu_s(q) = 0$$
 for all $q \neq q_0, q_1, \pi(q_1)$

$$\mu_{\rm S}(q_0) = \frac{\alpha_0 \kappa}{\delta(\kappa + \delta + \lambda) + \lambda \kappa} S$$

$$\mu_{\rm S}(q_1) = \frac{\alpha_1 \kappa}{\delta(\kappa + \delta + \lambda) + \lambda \kappa} S$$

$$\mu_{s}(\pi(q_{1})) = \frac{\lambda \kappa^{2}}{(\delta(\kappa + \delta + \lambda) + \lambda \kappa)(\delta + \kappa + \lambda)}S$$

• We must have
$$\pi(q_1) = \frac{\alpha_0 q_0 + \alpha_1 q_1}{\delta} = \bar{q}$$

HIGH PRICE SS EQUILIBRIUM

For individual rationality of $P(q_1)$ we must check: \rightarrow Gains from Trade

- Gains from trade
- No deviation to $P(q_0)$
- No deviation to $P(\pi(q_1))$

Trade off quality and liquidity

HIGH PRICE SS EQUILIBRIUM

- Let $d = u(q_1) u(q_0)$
- Can define thresholds on $\frac{x}{d}$ for each of the cases
- $\left(\frac{x}{d}\right)_h$ the maximum of the three cases
- $P(q_1)$ is equilibrium price iff $\frac{x}{d} \ge \left(\frac{\bar{x}}{d}\right)_h$

LOW PRICE SS EQUILIBRIUM

Sellers



- Positive mass only at q_0 and q_1
- Posterior is $\pi(q_0) = q_0$. SS Distribution

LOW PRICE SS EQUILIBRIUM

- Strict positive gains from trade $V_h(q_0) V_s(q_0) = \frac{X}{r+\delta+\kappa}$
- No deviation to $P(q_1)$:

$$\mathcal{M}(q_0) \left(V_h(q_0) - V_s(q_0) \right) \ge V_h(\pi(q_1)) - V_s(q_1)$$
• Find a threshold $\left(\bar{\frac{x}{d}} \right)_{\ell}$ such that we need $\frac{x}{d} \le \left(\bar{\frac{x}{d}} \right)_{\ell}$

EQUILIBRIUM REGIONS



 $κ = 0.1, λ = 0.2, r = 0.2, α_0 = 0.5, α_1 = 0.9$

Multiple equilibria region —> Strategic Complementarities

SUMMARY OF RESULTS

High Price Equilibrium

- High volume of trade and liquidity
- Buyers pay info. rent today, collect some of it in the future
- Pooling price, deteriorates beliefs and affects future market conditions

Low Price Equilibrium

- Low volume of trade; only pessimistic type trade
- No informational rent to any agent. Asset is "correctly priced"

SUMMARY OF RESULTS

Other Results

- Regions of Mixed strategy Eq. Appendix
- (Lemma) No other (pure strategy) SS equilibrium price can be sustained > Proof
- Comparative Statics
 Appendix
 - High frequency of re-trade ($\lambda \kappa$) shock harm high price equilibrium
 - Low search frictions, pure strategy eq. always exists

BELIEF DETERIORATION MECHANISM

• Assume $\alpha(q) = \delta \mu(q)$ for all q

Lemma

For any steady state in pure and symmetric strategies with μ_s as SS distribution of sellers and $q_{min} := \min_q \text{supp } \mu_s$, we cannot have any P(q) for $q > q_{min}$ as an equilibrium price.

- Only possible equilibrium candidate is one where only the most pessimistic trade
- Idea: after infinite cycles of re-trade, beliefs fully deteriorate
- Only bad news percolate in equilibrium
- Flipside of Camargo, Gerardi and Maestri (2019) in a sense

CONTRASTING WITH LEARNING-BY-HOLDING

- Learning-by-holding assumption (Hellwig and Zhang (2012), Guerrieri and Shimer (2014))
- Specialize $q_0 = 0$, $q_1 = 1$ and $\alpha_0 = \alpha_1 = \delta/2$
- Under high price eq, direct gains are the same as in the baseline.
- Different composition, different prob. of trade
- High price eq. cutoff

$$\left(\frac{x}{d}\right)^{l\,bh} := \frac{\delta(r+\delta+\kappa)}{r+\delta+\lambda}$$

CONTRASTING WITH LEARNING-BY-HOLDING

• Our high price equilibrium is more *fragile* when:

$$\left(\frac{\bar{x}}{d}\right)_{h} > \left(\frac{\bar{x}}{d}\right)^{l b h}$$

- True if $\kappa\lambda$ is large relative to δ
- $\delta \rightarrow 0 \Rightarrow$ high price is always equilibrium with learning-by-holding
- May not be sustainable with ex-post uninformed agents
- Intuition: information gets coarser
 - Mass of uninformed investors as a threat

EXTENSION: COSTLY LEARNING

- Buyers acquire learning rate θ at a cost $c(\theta)$
- Additional term in value functions (similar for sellers)

$$+\theta \left(qV_h(1,\theta) + (1-q)V_h(0,\theta) - V_h(q,\theta) \right)$$

- Choi (2018): Exog learning. Welfare is max with $\theta \to \infty$
- Endogenous θ : interaction Mkt Liquidity and Learning
- Our model delivers extreme result: No incentives to costly learn!
- Inefficiency: High price region larger for $\theta^* > 0$

FINAL REMARKS

- We drop the usual learning-by-holding assumption to model assets that are hard to verify
- Belief deterioration arises endogenously and is the central force determining price and liquidity
 - It hinders liquidity by making off-equilibrium threats more attractive for buyers
- Future work: asset design

Thank You!

Appendix

BASELINE MODEL

LAWS OF MOTION

Holders

$$\dot{\mu}_{ht}(q) = -\kappa \mu_{ht}(q) + \delta \mu_{st}(q) + \alpha(q)S + \lambda \left[\psi_t(\pi^{-1}(q)) \int_0^q d\mu_{st}(\tilde{q}) \right]$$
(3)

Sellers

$$\dot{\mu}_{st}(q) = \kappa \mu_{ht}(q) - \delta \mu_{st}(q) - \lambda \mu_{st}(q) \left[\int_{q}^{1} d\psi(\tilde{q}) \right]$$

Buyers

(4)

EQUILIBRIUM Definition

An **equilibrium** is a path of $\{(\psi_t(q))_{q \in [0,1]}\}, \mu_t(q), V_t(q) = (V_{st}(q), V_{ht}(q), V_{bt}), \text{ and an initial condition } \mu_0(q), \text{ such that, given the initial condition:}$

- i. Given $V_t(q)$ and $\mu_t(q)$, $\{\psi_t(q)\}_{q \in [0,1]}$ characterizes buyers' strategy in the stage game played at time $t, \forall t, q \in [0, 1]$;
- ii. Given $\{\psi_t(q)\}_{q\in[0,1]}, orall t, q\in[0,1], \mu(q)$ follows the LOM's;
- iii. Given $\{\psi_t(q)\}_{q \in [0,1]}, \forall t, V_t \text{ follows the value functions } \forall t, q \in [0,1].$
- iv. $(\mu_{st}(q), \mu_{ht}(q))_{q \in [0,1]}$ satisfies the resource constraints for all t
 - **SS equilibrium**, setting LOM's to zero.

HIGH PRICE SS EQUILIBRIUM

• LOM for beliefs q_i , i = 1, 2

$$\dot{\mu}_{h}(q_{i}) = -\kappa \mu_{h}(q_{i}) - \delta \mu_{h}(q_{i}) + \alpha_{i}S \xrightarrow{\text{S.S.}} \mu_{h}(q_{i}) = \frac{1}{\delta + \kappa} \alpha_{i}S$$

$$\dot{\mu}_{s}(q_{i}) = \kappa \mu_{h}(q_{i}) - \delta \mu_{s}(q_{i}) - \lambda \mu_{s}(q_{i}) \xrightarrow{\text{S.S.}} \mu_{h}(q_{i}) = \frac{\delta + \lambda}{\kappa} \mu_{s}(q_{i})$$

HIGH PRICE SS EQUILIBRIUM

• For q'_1 there is inflow via trade!

$$\dot{\mu}_h(q_1') = -\kappa \mu_h(q_1') - \delta \mu_h(q_1') + \lambda M_s(q_1) \xrightarrow{\text{S.S.}} \mu_h(q_1') = \frac{\lambda}{\delta + \kappa} M_s(1)$$

$$\dot{\mu}_{s}(q'_{1}) = \kappa \mu_{h}(q'_{1}) - \delta \mu_{s}(q'_{1}) - \lambda \mu_{s}(q'_{1}) \xrightarrow{\text{S.S.}} \mu_{h}(q'_{1}) = \frac{\delta + \lambda}{\kappa} \mu_{s}(q'_{1})$$

▶ Back

LOW PRICE EQ DISTRIBUTION

• From LOM we can get:

$$\mu_{s}(q_{0}) = \frac{\alpha_{0}\kappa(\delta + \kappa + \lambda)}{\alpha_{0}(\delta + \kappa + \lambda)^{2} + \alpha_{1}(\delta + \kappa)^{2}}S$$
$$\mu_{s}(q_{1}) = \frac{\alpha_{1}\kappa(\delta + \kappa)}{\alpha_{0}(\delta + \kappa + \lambda)^{2} + \alpha_{1}(\delta + \kappa)^{2}}S$$

▶ Back

SUMMARY OF RESULTS

- Some comparative statics (from simulations not analytically) > Graphs
- For High price equilibrium $\frac{x}{d} \ge \left(\frac{\bar{x}}{d}\right)_h$
 - i. \nearrow in α_0
 - ii. \searrow in α_1
 - iii. \searrow in λ
 - iv. \nearrow in κ
 - v. $\searrow in r$

SUMMARY OF RESULTS

- Some comparative statics (from simulations not analytically) > Graphs
- For Low price equilibrium $\frac{x}{d} \leq \left(\frac{\bar{x}}{d}\right)_{\ell}$
 - i. \nearrow in α_0
 - ii. \nearrow in α_1 (via δ)
 - iii. \searrow in λ
 - iv. \searrow in κ
 - v. $\searrow in r$

HIGH PRICE SS EQUILIBRIUM

• Direct gain from trade from deviating to any $P(q), q \leq q_1$

$$\begin{split} V_h(\pi(q)) - V_{\mathsf{S}}(q) &= \frac{1}{(r+\delta+\kappa)(r+\delta+\lambda)} \left[(\kappa+r+\delta+\lambda) \delta u(\pi(q)) - \right. \\ &\left. - \lambda \delta u(q_1) - (r+\delta+\kappa) \delta u(q) + x(r+\delta+\lambda) \right] \end{split}$$

- First term is negative and might be smaller for higher q
- But probability of trade increases with $q \longrightarrow$ **Trade off quality and liquidity**

▶ Back

NO OTHER SS EQUILIBRIUM PRICE

Lemma

There does not exist any other reservation price P(q) with $q \neq q_0, q_1$ that is an SS equilibrium price.

- By contradiction and follow SS mechanics
- Suppose $P(\hat{q})$ for $\hat{q} < q_0$ and $\hat{q} \neq q_1$
- But $q_0 \in \operatorname{supp} \mu_s$, so $\pi(\hat{q}) < \hat{q}$.
- So $\mu_h(\hat{q}) = 0 \Longrightarrow \mu_s(\hat{q}) = 0$, Deviation

NO OTHER SS EQUILIBRIUM PRICE

- Also $P(\hat{q})$ for $\hat{q} < q_0$ cannot be equilibrium prices.
- Would need that $\hat{q} = \min \operatorname{supp} \mu_{s}(q)$ and hence $\pi(\hat{q}) = \hat{q}$.
- LoM for holders and sellers in SS we have:

$$\dot{\mu}_{h}(\hat{q}) = -(\delta + \kappa)\mu_{h}(\hat{q}) + \lambda\mu_{s}(\hat{q}) \Longrightarrow \mu_{h}(\hat{q}) = \frac{\lambda}{\delta + \kappa}\mu_{s}(\hat{q})$$

$$\dot{\mu}_{s}(\hat{q}) = \kappa \mu_{h}(\hat{q}) - (\lambda + \delta +) \mu_{s}(\hat{q}) \Longrightarrow \mu_{h}(\hat{q}) = \frac{\lambda + \delta}{\kappa} \mu_{s}(\hat{q})$$

Both simultaneously can not hold

n BELIEF SUPPORT

- Case where replenish happens along $\{q_0, ..., q_N\}$ at rates $\{\alpha_0, ..., \alpha_N\}$
- $\sum_{i=1}^{N} \alpha_i = \delta$ and $\sum_{i=1}^{N} \alpha_i q_i = \delta \overline{q}_0$
- Posterior $P(q_N)$ is offered \bar{q}'_N
- The distribution

$$\mu_{S}(q_{i}) = \frac{\kappa \alpha_{i}}{(\kappa + \delta)(\delta + \lambda)} S \quad \forall i \in \{1, 2, ..., N\}$$
$$\mu_{S}(\bar{q}_{N}') = \frac{\lambda \kappa^{2}}{(\kappa + \delta + \lambda)(\kappa + \delta)(\delta + \lambda)} S \quad \forall i \in \{1, 2, ..., N\}$$

n BELIEF SUPPORT

- As *N* increases with δ fixed may have $\alpha_i \rightarrow 0$. Posterior dominates!
- (Conjecture) But then equilibrium region of high price shrinks, as economy goes to \bar{q}'_N

▶ Back

OLD INFO ACQUISTION

STAGE GAME

Informed Buyers

- We assume verification and its result are common knowledge, such that assets are traded by P_t(0) or P_t(1)
- Buyers' surplus after acquiring information:

$$\pi_t(1) \left(V_{ht}(1) - V_{st}(1) \right) + (1 - \pi_t(1)) \left(V_{ht}(0) - V_{st}(0) \right) - I$$

• Comparing the surplus of acquiring vs. not acquiring info we get a threshold l^* and the probability of checking asset's quality $\hat{\Phi}_t$

VALUE FUNCTIONS

Buyer

$$rV_{bt} = \lambda \left\{ \phi_t \Big[\pi(1) \big(V_{ht}(1) - V_{st}(1) \big) + (1 - \pi(1)) \big(V_{ht}(0) - V_{st}(0) \big) - I \Big] + \right\}$$

$$(1-\phi_t)\left[\int_0^1\psi_t(\tilde{q})\mathcal{M}(\tilde{q})\left(V_{ht}(\pi(\tilde{q}))-V_{st}(\tilde{q})\right)d\tilde{q}\right]\right\}+\dot{V}_{bt}$$

MIXED STRATEGY

MIXED STRATEGY EQUILIBRIUM

- Lemma rules out positive prob. on any $q \in [0,q_0) igcup (q_1,1]$
- SS argument as before.
- The max of support cannot be bigger than q_1
- The min of support cannot be smaller than q_0
- What happens in between? Not sure, but we rule out intervals!
- (Lemma): Any SS Mixed Strategy Eq σ is such that:

supp
$$\sigma = \{q_0, \pi^k(q_1), \pi^{k-1}(q_1), ..., \pi(q_1), q_1\}$$

for some *k* = 0, 1, 2, ...

MIXED STRATEGY EQUILIBRIUM



 $κ = 0.1, λ = 0.2, r = 0.9, α_0 = 0.5, α_1 = 0.9$

- Gap between two pure strategy equilibria \longrightarrow Mixed Strategy?

MIXED STRATEGY EQUILIBRIUM

- Construct one with supp $\sigma = \{q_0, q_1\}$
- Expression look a bit like before, shrinking λ
- Analytically cumbersome to check...
- Boils down to find σ_2 that equates gain of $P(q_1)$ and $P(q_0)$
- Some graphs → Link
- Still need to check deviations, and then construct the area of equilibrium...
- Mixed strategy might be weird at first, but in SS is an eq. with price dispersion

PROOF MIXED STRATEGIES Proposition

 $\sigma(q)$ = 0, $\forall q \in (q_1, 1]$

- Assume $\sigma(1) > 0$. Since $\pi(q') < q', \forall q' \in (0, 1]$ we have that $\pi(1) < 1$
- Also note that the replenish occurs only at $q \in \{q_0, q_1\}$. Therefore, we must have $\mu_h(1) = 0.$
- By LoM:

$$\dot{\mu_{s}}(1) = \kappa \mu_{h}(1) - \delta \mu_{s}(1) - \sigma(1)\mu_{s}(1) = 0 \Rightarrow \mu_{s}(1) = 0$$

• Update the maximum in the support of the beliefs and repeat this argument. This rules out all prices P(q) for $q \in (q_1, 1]$.

PROOF MIXED STRATEGIES Proposition

 $\sigma(q) = 0, \forall q \in [0, q_0)$

• We cannot follow the same argument since $\pi(0) = 0$. By LoM

$$\dot{\mu_h}(0) = -\mu_h(0)(\kappa + \delta) + \underbrace{\lambda\sigma(0)\mu_s(0)}_{\text{enter via trade}} = 0 \Longrightarrow \mu_h(0) = \frac{\lambda\sigma(0)\mu_s(0)}{\kappa + \delta}$$

$$\dot{\mu}_{s}(0) = \left(\lambda \underbrace{\int_{0}^{1} \sigma(\tilde{q}) d\tilde{q}}_{=1} - \delta\right) \mu_{s}(0) + \kappa \mu_{h}(0) = 0 \Longrightarrow \mu_{h}(0) = \frac{\delta + \lambda}{\kappa} \mu_{s}(0)$$

Therefore

CONTINUUM REPLENISH MODEL

CONTINUUM REPLENISH OF AGENTS

- δdt leave the model due maturity
- αdt enters following cdf F(q)
- Steps: Suppose $P(q^*)$ is price offered.
- Given $P(q^*)$ derive $G_s(q|q^*)$ and $G_h(q|q^*)$
- $P(q^*)$ will be eq price if:

$$q^* \in \operatorname{argmax}_{q \in [0,1]} \mathcal{M}(q|q^*) \left(V_h(\pi(q|q^*)|q^*) - V_{\mathcal{S}}(q|q^*) \right)$$

CONTINUUM REPLENISH OF AGENTS

- $\delta = \alpha$
- $\int_0^1 q dF(q) = \bar{q}_0$
- Endogenous distribution of sellers

$$G_{\mathsf{S}}(q|q^*) = \begin{cases} \frac{\delta\kappa}{(\delta+\kappa)(\delta+\lambda)}F(q) & \text{if } q < \pi(q^*) \\ \frac{\delta\kappa}{(\delta+\kappa)(\delta+\lambda)}F(q) + \frac{\lambda\kappa^2}{(\delta+\kappa+\lambda)(\delta+\kappa)(\delta+\lambda)}F(q^*) & \text{if } q \in [\pi(q^*), q^*) \\ \frac{\kappa}{\delta+\kappa}F(q) - \frac{\lambda\kappa}{(\delta+\kappa+\lambda)(\delta+\kappa)}F(q^*) & \text{if } q \ge q^* \end{cases}$$

CONTINUUM REPLENISH OF AGENTS

• FOC for buyer's problem:

$$\Theta f(q^*) \left(\frac{\delta \kappa d}{(r_h r_s)} (\pi(q^*) - q^*) + \frac{x}{r_s} \right) - (\Theta + \lambda \kappa) F(q^*) \frac{\delta d}{r} \left(1 - \frac{q^* f(q^*)}{\Gamma(1|q^*)} \right) = 0$$
(5)

• Sufficient condition for (local) max is f(1) small enough:

$$\bar{q} > f(1) \frac{(\delta + \kappa + \lambda)}{\kappa}$$

• Single crossing? Multiplicity?

EXAMPLE

• F(q) = q(2 - q)

