MANAGER ALLOCATION AND FIRM SIZE

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MOTIVATION

- Finding, training and retaining managerial talent is a key determinant of firm productivity

- How do firms of different sizes deal with managerial turnover?
  - Larger firms, active internal markets, larger sample of talent to choose from
  - Smaller firms, restricted pool of talent, have to rely on external hires

- How do these forces affect firms' size and productivity?

- How do they shape the distribution of firm productivity?

THIS PROJECT

- Study managerial allocation and turnover across firms of different sizes

- Data

  - German LIAB employer-employee administrative data
  - Occupation details inside firms (5-digit level)
  - Today: Evidence of flows across firm size distribution; event study of managerial hires.

- Model

  - Firm Dynamics and Search Frictions on the managerial positions
  - Today: Simplified firm decision problem
  - Understand manager turnover vis-a-vis firm's composition (size) choice

SOME LITERATURE

- Managerial Alloc. and Productivity: Bloom and Van Reenen (2007), Minni (2023), Friedrich (2023), Pastorino (2022), Metcalfe, Sollaci, and Syverson (2023), Bender et al. (2018)

- Firm hierarchy and task division: Garicano and Rossi-Hansberg (2006), Caliendo et al. (2020), Adenbaum (2023), Kohlhepp (2023), Freund (2024)

- Firm dynamics with frictional labor markets: Schaal (2017), Gouin-Bonenfant (2022), Bilal et al. (2022), Herkenhoff et al. (2018), Elsby and Gottfries (2022), Audoly (2023)

DATA

DATA SOURCES

- LIAB - Worker-level data ‹ Cleaning
    - Representative sample of establishments surveyed from 2009–2016
    - *Entire workforce* of these firms is recorded (panel cases)
    - Entire biographies of these workers from 1975–2019
    - Spell-level data on daily wages, occupation, and matched firm characteristics
- BHP - Establishment-level data
    - 50% sample of all establishments in Germany
    - Granular industry data (5 digits), district location
    - Annual Data on Occupation and Wage Structure
    - Extensions on firm inflows/outflows, entry and exit
    - All these components can be matched to LIAB

MANAGERS IN THE DATA

- 5-digit occupation codes (*KLdB 2010*)
    - First 3 digits give an overall occupation
    - 4th digit tells us if it is a manager!
    - 5th digit separates the complexity of the task into 4 levels
- For managers
    - 3 Complex Task: "Supervisor"
    - 4 Highly Complex Task: "Manager"
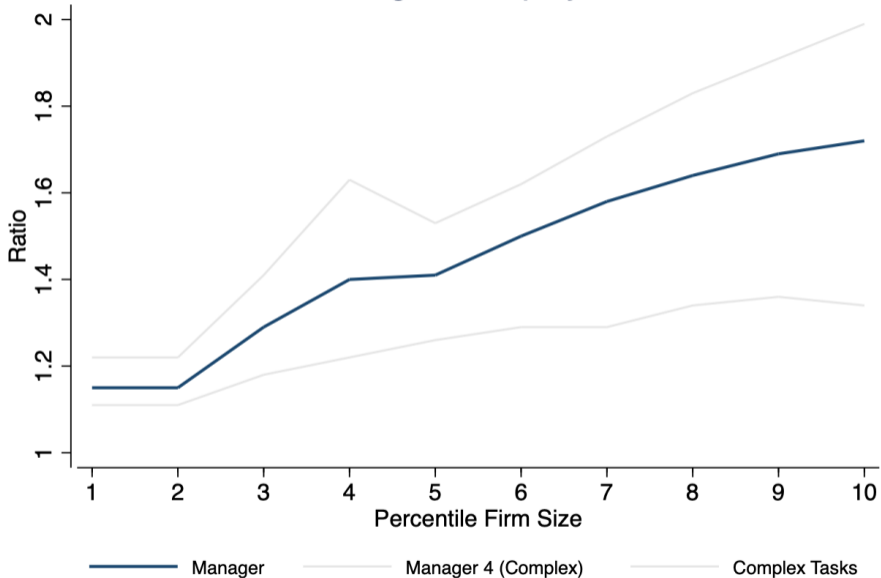- It is more granular than ISCO-08

ARE MANAGERS RELEVANT IN THE DATA?

- Is it economically meaningful to split the data into managers and non-managers?
- Or it is just "labeling"?
- Can we disentangle from high-complex tasks?
- Let's look at some numbers
    1. Firm size and Wage
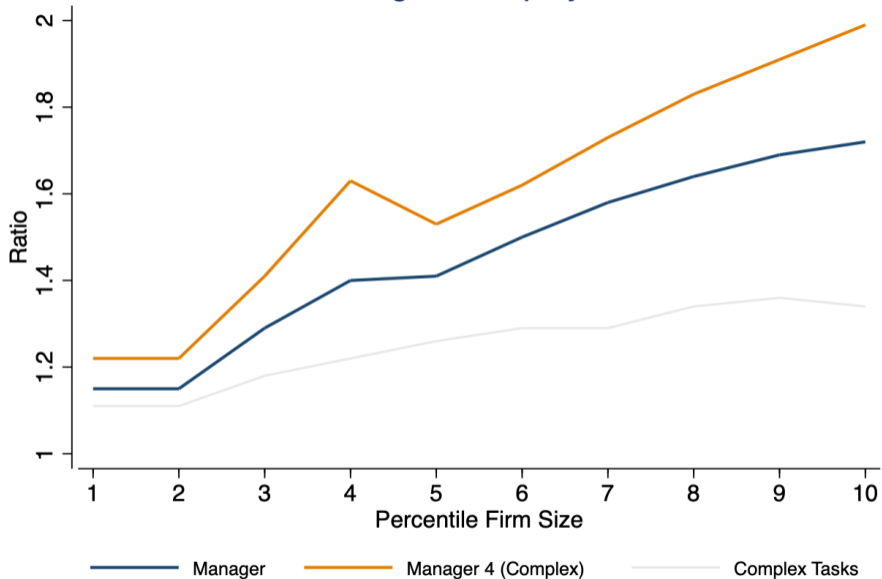   2. Internal and external flows of managers

FIRM SIZE DISTRIBUTION CONDITIONAL ON MANAGER

	Firm w/o Manager	Firm with Manager
Mean	**22.11**	**93.44**
STD	**47.69**	**103.07**
p5	2	5
p10	2	9
Median	8	52
p90	46	250
p95	82	328

‹ Employment Share   ‹ Ratio Wage-Employment

Ratios of Wage to Employment Share

Ratio vs. Percentile Firm Size

Legend: Manager — Manager 4 (Complex) — Complex Tasks

Ratios of Wage to Employment Share

Manager — Manager 4 (Complex) — Complex Tasks
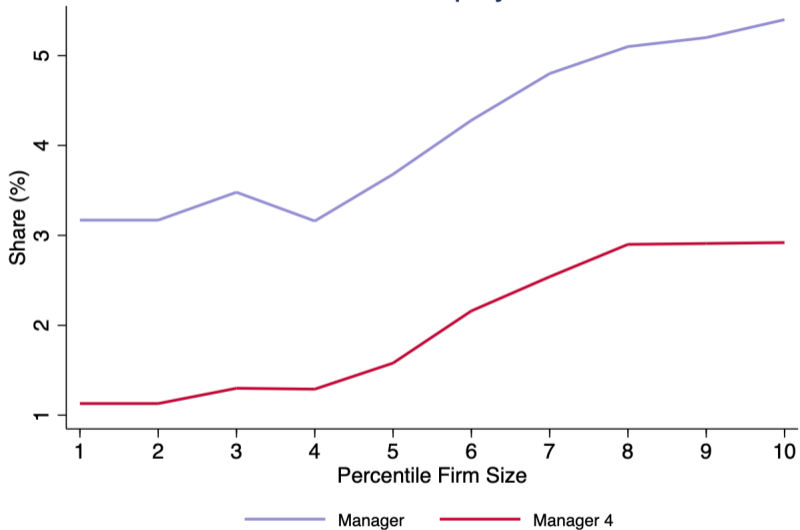
Ratios of Wage to Employment Share
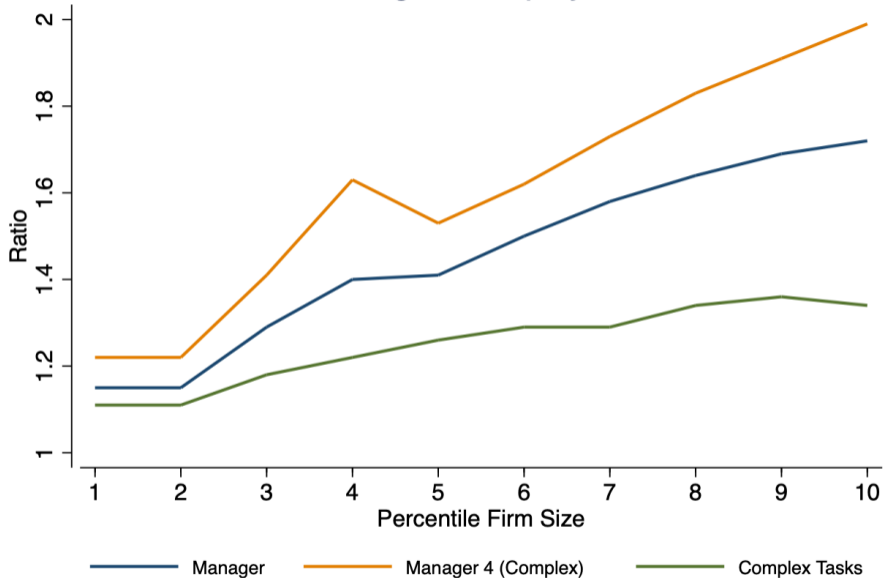
Share of Employment

RATIOS OF WAGE AND EMPLOYMENT SHARE

Sample	Ratio Managers	Ratio Managers 4
Whole	1.54	1.73
Cond. on Man.	1.53	1.73
Cond. on Man. 4	1.63	1.72
Q. Size 1	1.12	1.13
Q. Size 2	1.38	1.56
Q. Size 3	1.56	1.72
Q. Size 4	1.69	1.92

Establishment Size Distribution (Conditional on Manager)

Percentile	Estb. Size
1%	11
5%	12
10%	14
25%	22
50%	43
75%	117
95%	524
Mean	**171.85**
STD	**1311.88**

INFLOWS INTO MANAGERS AND NON-MANAGERS

From	Into Man	Into Non-Man
Man same firm	7.50%	1.02%
Man diff firm	15.80%	1.02%
Non-Man same firm	36.13%	12.70%
Non-Man diff firm	21.86%	41.50%
Unemp.	18.50%	43.60%

‹ Outflows   ‹ Share of Total Workers

FLOW INTO MANAGERS, BY SIZE QUARTILE
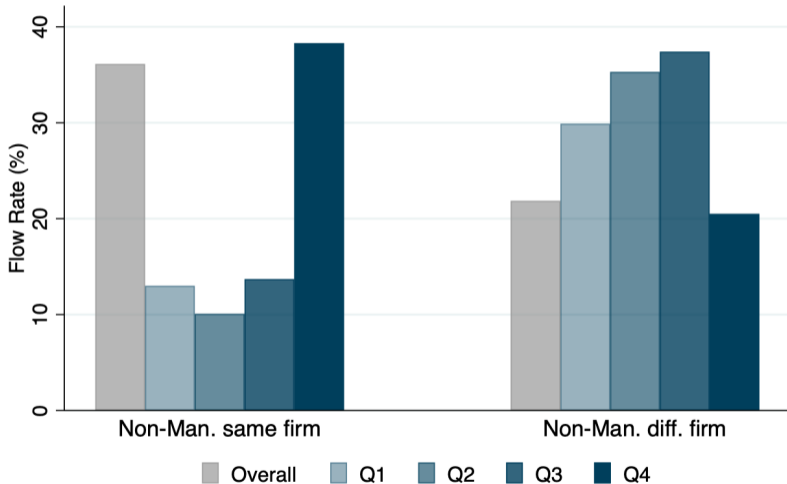
FLOW INTO NON-MANAGERS, BY SIZE QUARTILE
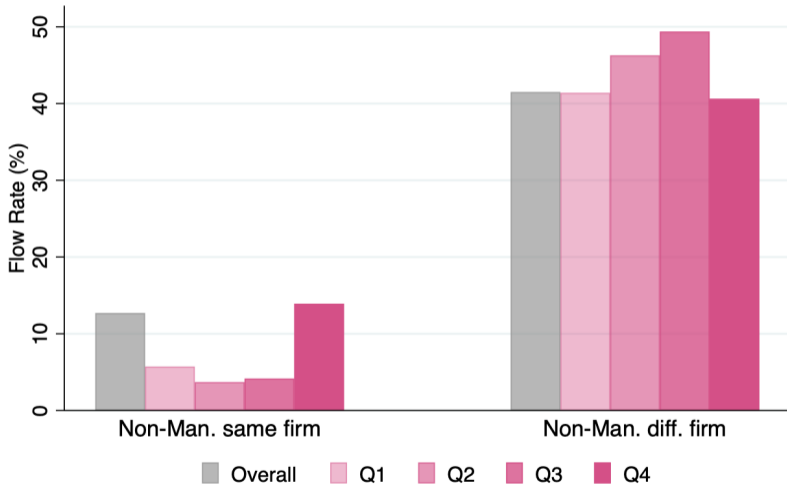
INTERNAL LABOR MARKET FOR MANAGERS

- Larger firms have a higher share of *its own inflows* coming from internal hiring

- Small and medium firms rely more on external hires for managers

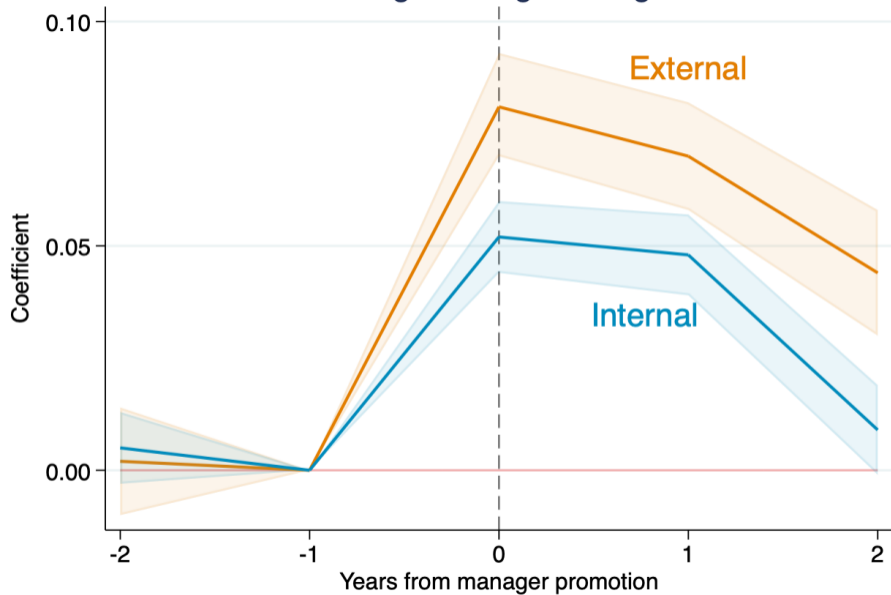- Pattern for non-manager hiring is more homogenous across firm sizes

INTERNAL AND EXTERNAL MANAGER HIRES

- Simple event study regression
- Understand what is happening at the worker level upon an internal and external hire

$$y_{it} = \sum_{h \in \{-2,\ldots,2\}} \left(\gamma_h \cdot D^I_{i,t-h} + \beta_h \cdot D^E_{i,t-h} \right) + \alpha_i + \delta_t + \epsilon_{jt}$$

- y_{it}: Log Wage
- Ind, and time fixed effects + controls(educ, occupation, industry)
- $D^I_{i,t-h}$: Dummy for h years away from internal manager hire
- $D^E_{i,t-h}$: Dummy for h years away from external manager hire

FLOWS AND FIRM BEHAVIOR

- Simple event study regression
- Understand what is happening at the firm level upon an internal and external manager hire

$$y_{jt} = \sum_{h \in \{-2,...,2\}} \left(\gamma_h \cdot D^I_{j,t-h} + \beta_h \cdot D^E_{j,t-h} \right) + \alpha_j + \delta_t + \epsilon_{jt}$$

- y_{jt}: (log) Size, Wage Bill and Average Wage
- Specification that considers the *only non-managerial side* of the firm
- $D^I_{j,t-h}$: Dummy for *h* years away from internal manager hire
- $D^E_{j,t-h}$: Dummy for *h* years away from external manager hire

Manager Hiring on Firm size

Internal

External

Coefficient

Years from manager hiring

Manager Hiring on Wage Bill

Manager Hiring on Average Wage

MODEL

MODEL SETTING

- Time is continuous, discounted at r
- Firm has fixed productivity a
- Firm can hire 1 *manager*, and *n workers*
- Manager of quality $z \in [\underline{z}, \overline{z}]$ and suffer shocks acc. to cfd $G(z)$, arrival rate γ

$$y(z, n, a) = azn^{\alpha}, \quad \alpha \in (0, 1)$$

- Firm with no managers

$$y_0(n, a) = an^{\alpha}$$

MANAGERS: EXTERNAL LABOR MARKET

- Frictional inflows and outflows of managers to the Firm

- Let **X** be the (abstract) relevant state

- $\delta(\mathbf{X})$ effective rate of separations
    - Exogenous job destruction
    - Endogenous quits towards poaching firms

- $\lambda_e(\mathbf{X})$ effective rate of external hires
    - From unemployment pool
    - From other firms
    - Upon meeting, face dist. $\Gamma\left(\widetilde{z}|\mathbf{X}\right)$

MANAGERS: INTERNAL LABOR MARKET

- From the existing pool of workers, promote to manager

- Technology to promote is a rate of arrival $\lambda_i(a, n)$

- Increasing in both a and n

- If find a manager internally $\longrightarrow \bar{z}$

- Conditional on continuing in the firm, flow payment $w(z, n, a)$
- For now: Silent about its determination; assume known to the Firm
- Later: In a full model this will come from some form of bargaining
  - For instance surplus sharing

$$w(z, n, a) = \text{argmax}[J(z, n, a) - J_0(n, a)]^{\theta}[W(z, n, a) - U(z)]^{1-\theta}$$

  - Maybe fancier stuff like Elsby and Gottfries (2022) or Audoly (2023)
- Intuitively: Makes sense to be increasing in z and a; up to a point in n

WORKERS: COMPETITIVE LABOR MARKET

- Firm can hire *n* workers on spot competitive market

- Wage rate *w* given to the firm

- Firm incurs adjustment cost on hiring and firing

- Let $\dot{n} = h$ the incremental hires/fires

$$c(h) = \frac{ch^{\phi}}{\phi}$$

- Think of 2 cases:
  - $\phi = 1$: Linear adjustment costs
  - $\phi = 2$: Convex adjustment costs

VALUE FUNCTIONS

- Value for the firm with manager z, n workers, and productivity a

$$rJ(z, n, a, \mathbf{X}) = \max_{h \in \mathbb{R}} \left[y(z, n, a) - wn - w(z, n, a) \right.$$

$$+ \delta(\mathbf{X})(J_0(n, a) - J(z, n, a))$$

$$+ \gamma \left(\int \max \left\{ J(\tilde{z}, n, a), J_0(n, a) \right\} - J(z, n, a) dG(\tilde{z}) \right)$$

$$\left. - c(h) + J_n(z, n, a, \mathbf{X}) \cdot h \right]$$

VALUE FUNCTIONS

- Value for the firm with no manager, n workers, and productivity a

$$rJ_0(n, a, \mathbf{X}) = \max_{h_0 \in \mathbb{R}} \Bigg[y_0(n, a) - wn$$

$$+ \lambda_i(a, n) \Big(J(\bar{z}, n, a) - J_0(n, a) \Big)$$

$$+ \lambda_e(\mathbf{X}) \left(\int \max \Big\{ J(\tilde{z}, n, a) - J_0(n, a), 0 \Big\} d\Gamma(\tilde{z}|\mathbf{X}) \right)$$

$$- c(h_0) + J_{0n}(n, a, \mathbf{X}) \cdot h_0 \Bigg]$$

- In a full model these are equilibrium objects

- Depend on firms (n, a) ability to keep and poach managers *relative* to the distribution of other firms

    – Depends on where the firm is in the job ladder

- Changing *n* potentially affect your relative position (Elsby and Gottfries (2022))

WHAT ARE $\delta(\mathbf{X})$ AND $\lambda_e(\mathbf{X})$?

- Interesting point: How do different firms manage turnover?

- Three forces at play on the choice of n:

    – Relative position in the distribution of (z, n, a) firms

    – Chances of internal promotion

    – Departures from optimal size and adjustments costs

- How do these forces shape the distribution of firm productivity and size?

- Today: Take the firm point of view and assume "reduced form"

    – $\delta(\mathbf{X}) = \delta(n, a)$ decreasing in n and a
    – $\lambda_e(\mathbf{X}) = \lambda_e(n, a)$ increasing in n and a

FOCS AND THRESHOLD

- Omit a for ease of notation

- Simple FOCs, with policy functions $h(z, n)$ and $h_0(n)$

$$J_n(z, n) = c'(h(z, n))$$

$$J_{0n}(n) = c'(h_0(n))$$

- From the max operators, keep/hire iff

$$z \geq z(n) \text{ with } J(z(n), n) = J_0(n)$$

$$J(z, n) = \beta(n) \left[\pi(z, n) + p(n)J_0(n) + \gamma \int_{z(n)}^{\overline{z}} J(\tilde{z}, n)dG(\tilde{z}) \right.$$

$$\left. - c(h(z, n)) + J_n(z, n)h(z, n) \right]$$

- $\beta(n) = (r + \delta(n) + \gamma G(z(n)))^{-1}$ effective discount rate
- $p(n) = \delta(n) + \gamma G(z(n))$ effective separation rate

ENVELOPE CONDITIONS

$$J_n(n, z) = \beta(n)\left[\pi'(z, n) + \delta'(n)(J_0(n) - J(z, n)) + \Big(\delta(n) + \gamma G(z(n))\Big)J_{0,n}(n)\right.$$

$$+ \gamma \int_{z(n)}^{\overline{z}} J_n(\tilde{z}, n)dG(\tilde{z})$$

$$\left. -c'(h(z, n))h'(z, n) + J_{nn}(z, n)h(z, n) + J_n(z, n)h'(z, n)\right]$$

$$J_n(n, z) = \beta(n)\left[\pi'(z, n) + \delta'(n)(J_0(n) - J(z, n)) + \Big(\delta(n) + \gamma G(z(n))\Big)J_{0,n}(n)\right.$$

FIRMS WITH NO MANAGER

- Similar steps as before

$$c'(h_0(n)) = \beta_0(n)\left[\pi'_0(n) + \lambda'_i(n)(J(\bar{z}, n) - J_0(n)) + \lambda_i(n)c'(h(\bar{z}, n))\right.$$

$$+ \lambda'_e(n)\left[-(1 - \Gamma(z(n)))J_0(n) + \int_{z(n)}^{\bar{z}} J(\tilde{z}, n)d\Gamma(\tilde{z})\right]$$

$$+ \lambda_e(n)\int_{z(n)}^{\bar{z}} c'(h(\tilde{z}, n))d\Gamma(\tilde{z})$$

$$+ (\phi - 1)c'(h_0(n))h'_0(n)\Bigg]$$

SANITY CHECK: LINEAR CASE

- Consider the linear case $c(h) = ch$ (or $\phi = 1$)

- Naturally n is not a state in this case! $\longrightarrow J(z)$ and J_0

- The threshold is $z(n) = \hat{z}$ for all n

- Choice of n is static but takes into account the effect on transition rates

SANITY CHECK: LINEAR CASE

$$\delta'(n^*(z))\underbrace{\left(J(z) - J_0\right)}_{=S(z)} = \pi'(z, n^*(z)) - rc$$

- Given the VFs, this pins down $n^*(z)$ optimal worker's level given z
- Firm *over-hires* compared to a model with no job ladder

SANITY CHECK: LINEAR CASE

- Similar logic for the firm with no manager

$$rc = \pi_0'(n_0^*) + \lambda_i'(n_0^*) \left(\frac{\pi'(\bar{z}, n^*(\bar{z})) - rc}{\delta'(n^*(\bar{z}))} \right) + \lambda_e'(n_0^*) \int_{\hat{z}}^{\bar{z}} \frac{\pi'(\tilde{z}, n^*(\tilde{z})) - rc}{\delta'(n^*(\tilde{z}))} d\Gamma(\tilde{z})$$

- n_0^* takes into account the marginal contribution of the firm's size to transition rates

- Given $\{S(z)\}$ and $\hat{z} \implies n^*(z)$ and n^*

- Check if VFs and threshold condition holds

WHAT I SHOWED YOU TODAY

- Detailed occupational data from Germany

  – Identify managers

  – Sense of firm composition for occupations and task complexity

- Managers are roughly who we think they are

  – Better paid and in larger firms

- Internal labor markets for managers are relevant

- Model that allows for between and within firm flows of managers

MODEL NEXT STEPS

- Cast the model in general equilibrium
  - Manager's problem will pin down the flows between firms
  - Internal promotion affects the margins of keeping and poaching managers
  - Interesting to see the equilibrium implications of these forces to the firm distribution
- This model accommodates more layers of competitive workers
  - $y(z, n_1, n_2) = az(n_1^\alpha n_2^{1-\alpha})^\nu$
  - Promotion tech. might depend on the pool of a specific type of worker inside the firm
    (need more evidence on this)
- Is this the relevant way to think about internal labor markets?

DATA NEXT STEPS

- Many possibilities…

- AKM-like approach to assess the quality of firms and managers

- Maybe relevant analysis is at the team level (Jarosch, Oberfield, and Rossi-Hansberg (2021))

  – Inside the firm, workers that share the 3-digit occupation

  – Depict some hierarchy using managers and complexity levels

  – Connect to Garicano and Rossi-Hansberg (2006)

Thank You!

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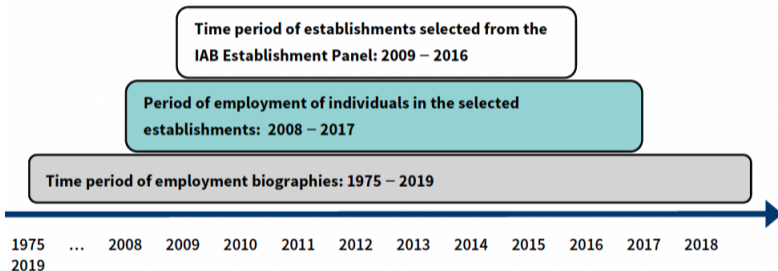
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DATA CLEANING

- Cleaning process following Dauth and Eppelsheimer (2020)
- Merge with BHP (firm data)
- Generate Industries 2 digit, Occ and educational consolidation groups
- Wages: Deflated to 2015 Euros
- Flagging top-censored wages
- Have available an imputed wage following Card, Heining, and Kline (2013), Dustmann, Ludsteck, and Schönberg (2009)
- Yearly panel with main episode and overlapping Jan 31st (spell-level data also available)
- Construct *panel cases* from establishments that were surveyed by the, where we have the entire workforce of the firm

DATA SOURCES

- LIAB LM 1975-2019

Time period of establishments selected from the
IAB Establishment Panel: 2009 – 2016

Period of employment of individuals in the selected
establishments: 2008 – 2017

Time period of employment biographies: 1975 – 2019

1975 … 2008 2009 2010 2011 2012 2013 2014 2015 2016 2017 2018
2019

SOME NUMBERS ON THE PANEL CASES

- Overall
  - From 2009–2016, ~ 7k firms, ~ 600k workers per year
  - 2-digit Industry: Manufacturing (23%), Trade & Repair (15%), Real Estate (12%), Construction (8%)

- Managers
  - ~ 35k managers per year, around 5% of the workforce
  - 2-digit Industry: Manufacturing (25%), Trade & Repair (17%), Real Estate (11%), Construction (9%)
  - Manager + High Complexity: ~ 20k per year (after 2011)
  - Similar pattern of industries, but more heavily concentrated in healthcare and education

‹ Back

EXAMPLE KLDB 2010 AND ISCO-08 COMPARISON

KldB 2010 (5-Digit)	KldB 2010 Classification title	ISCO-08 (4-Digit)	ISCO Unit Group
27394	Managers in production planning and scheduling	1321	Manufacturing managers
28194	Managers in textile making	1321	Manufacturing managers
28294	Managers in the production of clothing and other textile products	1321	Manufacturing managers
28394	Managers in leather- and fur-making and -processing	1321	Manufacturing managers
29194	Managers in beverages production	1321	Manufacturing managers
29294	Managers in the production of foodstuffs (etc)	1321	Manufacturing managers
82594	Managers in medicine, orthopaedic and rehabilitation technology	1321	Manufacturing managers

EXAMPLE KLDB 2010 AND ISCO-08 COMPARISON

| KldB 2010 (5-Digit)	Classification title	ISCO-08 (4-Digit)	ISCO-08 (4-Digit)
29301	Cooks (without specialization)-unskilled/semiskilled tasks	9412	Kitchen helpers
29302	Cooks (without specialization)-skilled tasks	5120	Cooks
29312	Hors d'œuvrier, pantry or pastry cooks-skilled tasks	5120	Cooks
29322	Roast, grill or fish cooks-skilled tasks	5120	Cooks
29382	Cooks (with specialization, not elsewhere classified)-skilled tasks	5120	Cooks
29393	Supervisors in cooking	3434	Chefs
29394	Managers in cooking	3434	Chefs

MANAGERS ON THE PANEL CASES

Category	2009	2010	2011	2012	2013	2014	2015	2016	Total
Non-Managers	593,817	582,362	571,662	573,434	579,256	589,614	610,520	626,717	4,727,382
Managers	22,101	22,666	31,192	37,441	37,745	37,694	37,906	38,533	265,278
(%)	3.72%	3.89%	5.46%	6.53%	6.52%	6.39%	6.21%	6.15%	5.61%

FIRMS WITH AT LEAST ONE MANAGER

Category	2009	2010	2011	2012	2013	2014	2015	2016	Total
Firms w/o Managers	4,221	4,562	4,150	4,117	4,360	4,673	5,077	5,671	36,831
Firm with Managers	1,945	1,941	2,445	2,646	2,712	2,780	2,864	2,976	20,309
Percentage (%)	46.08%	42.55%	58.92%	64.27%	62.20%	59.49%	56.41%	52.48%	55.14%

Share of Employment
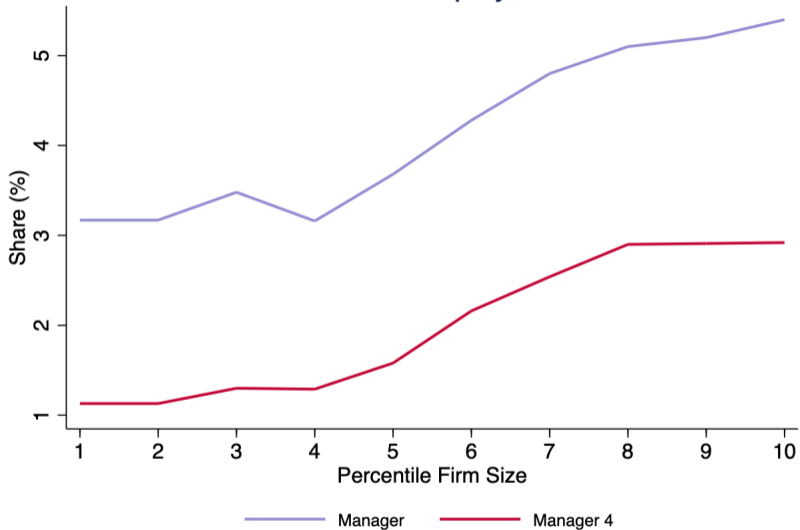
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Q. Size 3	1.56	1.72
Q. Size 4	1.69	1.92

OUTFLOWS

OUTFLOWS OUT OF MANAGERS AND NON-MANAGERS

| To	Out of Man	Out of Non-Man
Man same firm	10.80%	1.79%
Man diff firm	20.16%	1.12%
Non-Man same firm	21.07%	13.45%
Non-Man diff firm	20.00%	43.22%
Unemp.	27.80%	40.37%

ARE MANAGERS SPECIAL?

- Demotions and transitions to non-management are of the same strength (20%)
- But wages might tell a different story (not sure if that is the focus)

FLOW OUT OF MANAGERS, BY SIZE QUARTILE

FLOW OUT OF NON-MANAGERS, BY SIZE QUARTILE

Legend: ■ Overall ■ Q1 ■ Q2 ■ Q3 ■ Q4
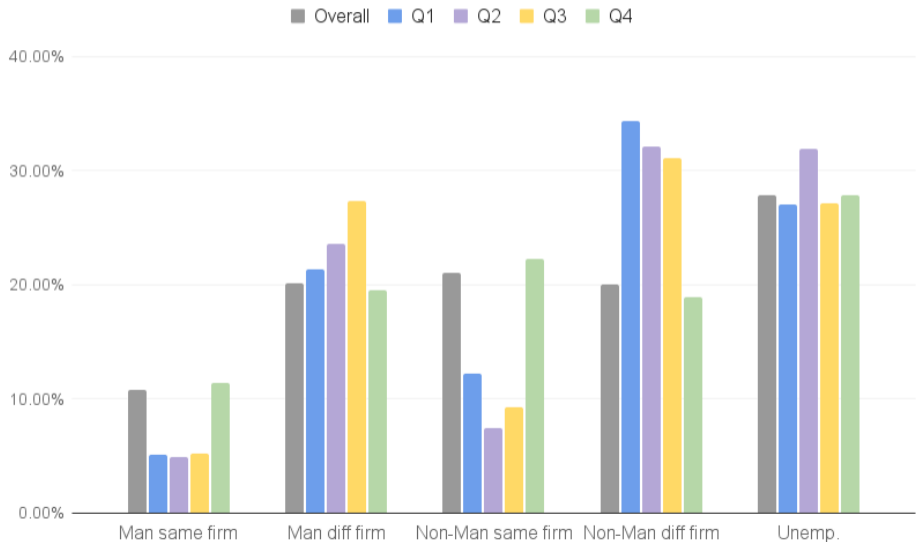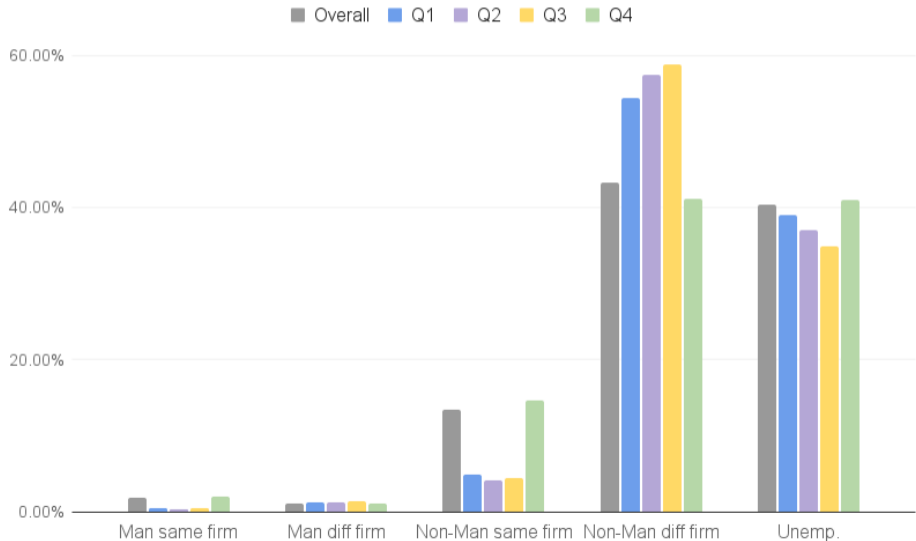
FLOWS AS SHARES OF TOTAL WORKERS

INFLOWS INTO MANAGERS AND NON-MANAGERS (TOTAL WORKERS)

From	Into Man.	Into Non-Man.	Unemp.
Man. same firm	0.90%	0.17%	0.79%
Man. diff firm	2.10%	0.17%	
Non-Man. same firm	4.70%	2.10%	1.20%
Non-Man. diff firm	2.80%	7.10%	
Unemp.	2.40%	7.50%	-

ARE MANAGERS SPECIAL?

- Single largest inflow into management is the internal market (4.7%)
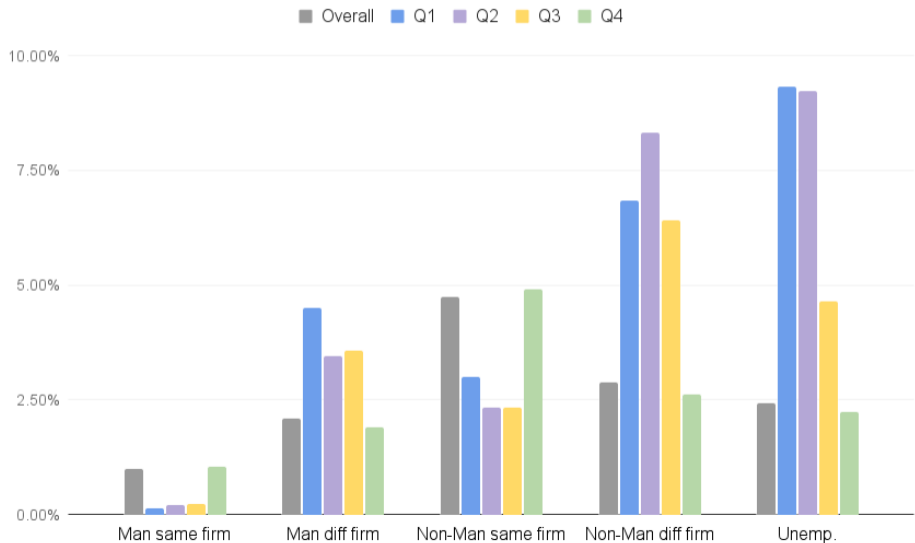
- Internal movement of non-managers is smaller (2.1%)

ARE MANAGERS SPECIAL?

- Somewhat easier to bring a non-manger of another firm to become your manager (2.8%)…
- …than bring an already manager from another firm (2.1%)
- Demotions and transitions to non-management are of the same strength (0.17%)
- But wages might tell a different story (not sure if that is the focus)
  - Relative to the pool of managers this flow is 2.8% of the mass of managers
- Mangers are also less likely to come from (2.4%) and to unemployment (0.79%) compared to non-managers (7.5% and 1.2%)

OUTFLOWS OUT OF MANAGERS AND NON-MANAGERS

To	Out of Man	Out of Non-Man	Out of Unemp
Man same firm	0.99%	0.29%	2.43%
Man diff firm	2.09%	0.18%	
Non-Man same firm	2.80%	2.19%	7.53%
Non-Man diff firm	2.82%	7.16%	
Unemp.	0.79%	1.22%	

FLOW INTO MANAGERS, BY SIZE QUARTILE (TOTAL WORKERS)

Legend: Overall, Q1, Q2, Q3, Q4

	Man same firm	Man diff firm	Non-Man same firm	Non-Man diff firm	Unemp.
Overall	1.0%	2.1%	4.7%	2.9%	2.4%
Q1	0.1%	4.5%	3.0%	6.8%	9.3%
Q2	0.2%	3.4%	2.3%	8.3%	9.2%
Q3	0.3%	3.5%	2.3%	6.4%	4.6%
Q4	1.0%	1.9%	4.9%	2.6%	2.2%

FLOW INTO NON-MANAGERS, BY SIZE QUARTILE (TOTAL WORKERS)

Legend: Overall, Q1, Q2, Q3, Q4

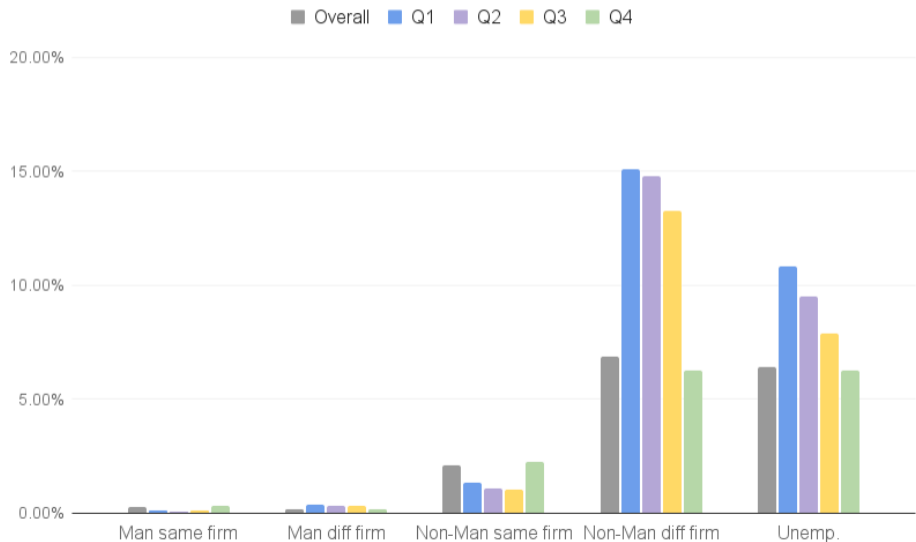| | Man same firm | Man diff firm | Non-Man same firm | Non-Man diff firm | Unemp. |
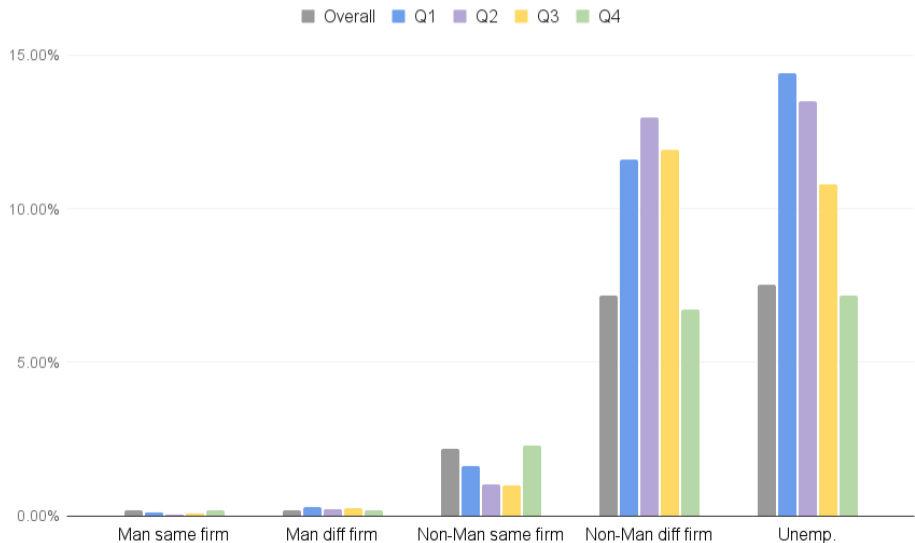
INFLOWS BY SIZE QUARTILE

- Bigger firms have a stronger internal labor market for Managers
- Middle firms rely on other firms managers (especially the small ones)
- Also on non-managers from other firms
- Consistent with a story of firm size complexity of the production
- Moreover the 4th quartile seems to dictates the average strength of the flows
    - It is where most transitions occur

FLOW OUT OF MANAGERS, BY SIZE QUARTILE (TOTAL WORKERS)
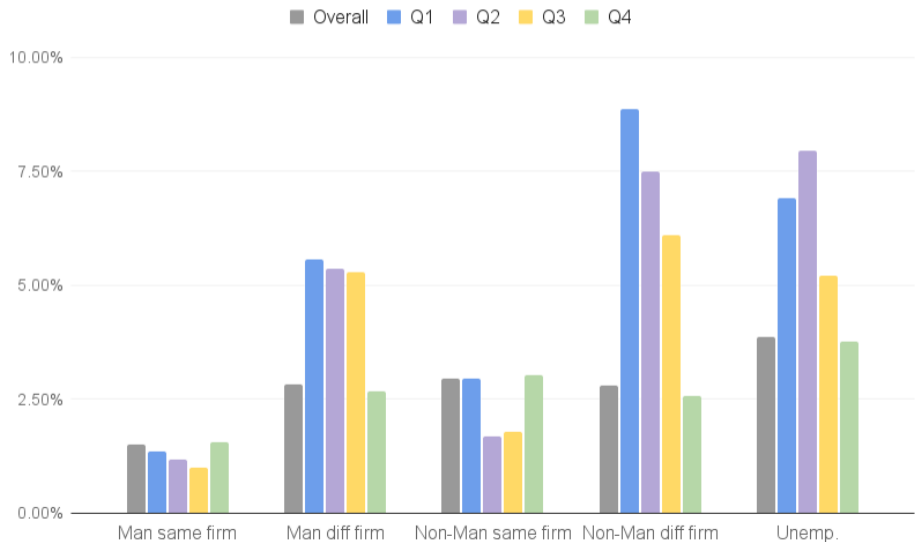
FLOW OUT OF NON-MANAGERS, BY SIZE QUARTILE (TOTAL WORKERS)

Legend: Overall, Q1, Q2, Q3, Q4

Y-axis: 0.00%, 5.00%, 10.00%, 15.00%, 20.00%

X-axis categories: Man same firm, Man diff firm, Non-Man same firm, Non-Man diff firm, Unemp.

From	Into Man	Into Non-Man
Man same firm	0.15%	0.11%
Man diff firm	4.70%	0.29%
Non-Man same firm	3.10%	1.59%
Non_Man diff firm	6.73%	11.60%
Unemp.	9.47%	14.40%

2ND QUARTILE

From	Into Man	Into Non-Man
Man same firm	0.18%	0.06%
Man diff firm	3.41%	0.23%
Non-Man same firm	2.39%	1.02%
Non-Man diff firm	8.23%	13.05%
Unemp.	9.13%	13.50%

3RD QUARTILE

From	Into Man	Into Non-Man
Man same firm	0.21%	0.07%
Man diff firm	3.56%	0.25%
Non-Man same firm	2.32%	0.99%
Non_Man diff firm	6.40%	11.90%
Unemp.	4.64%	10.80%

From	Into Man	Into Non-Man
Man same firm	1.04%	0.18%
Man diff firm	2.00%	0.17%
Non-Man same firm	4.90%	2.29%
Non-Man diff firm	2.63%	6.73%
Unemp.	2.23%	7.16%

FIRMS WITH NO MANAGER

$$J_0(n) = \beta_0(n)\left[\pi_0(n) + \lambda_i(n)J(\bar{z}, n) + \lambda_e(n)\int_{z(n)}^{\bar{z}} J(\tilde{z}, n)d\Gamma(\tilde{z})\right.$$

$$\left. - c(h_0(n)) + J_{0n}(n)h_0(n)\right]$$

- $\beta_0(n) = \left(r + \lambda_i(n) + [1 - \Gamma(z(n))]\lambda_e(n)\right)^{-1}$ effective discount rate

ENVELOPE CONDITIONS

- For the firm with no managers

$$c'(h_0(n)) = \beta_0(n)\left[\pi'_0(n) + \lambda'_i(n)(J(\bar{z}, n) - J_0(n)) + \lambda_i(n)c'(h(\bar{z}, n)) \right.$$

$$\lambda'_e(n)\left[-(1 - \Gamma(z(n)))J_0(n) + \int_{z(n)}^{\bar{z}} J(\tilde{z}, n)d\Gamma(\tilde{z}) \right]$$

$$+ \lambda_e(n)\int_{z(n)}^{\bar{z}} c'(h(\tilde{z}, n))d\Gamma(\tilde{z})$$

$$\left. + (\phi - 1)c'(h_0(n))h'_0(n) \right]$$

SANITY CHECK: LINEAR CASE

- Back to VF's at the optimal policy

$$c'(h(z,n)) = \beta(n)\left[\pi'(z,n) + \delta'(n)\Big(J_0(n) - J(z,n)\Big) + \Big(\delta(n) + \gamma G(z(n))\Big)c'(h_0(n))\right.$$

$$+ \gamma \int_{z(n)}^{\overline{z}} c'(h(\widetilde{z},n))dG(\widetilde{z})$$

$$\left. + (\phi - 1)c'(h(z,n))h'(z,n)\right]$$

- Back to VF's at the optimal policy

$$c'(h(z,n)) = \beta(n)\left[\pi'(z,n) + \delta'(n)\Big(J_0(n) - J(z,n)\Big) + \Big(\delta(n) + \gamma G(\hat{z})\Big)c'(h_0(n)) \right.$$

$$+ \gamma \int_{\hat{z}}^{\overline{z}} c'(h(\widetilde{z},n))dG(\widetilde{z})$$

$$+ \left. (\phi - 1)c'(h(z,n))h'(z,n) \right]$$

- Back to VF's at the optimal policy

$$c = \beta(n)\left[\pi'(z,n) + \delta'(n)\Big(J_0 - J(z)\Big) + \Big(\delta(n) + \gamma G(\hat{z})\Big)c + \gamma \int_{\hat{z}}^{\overline{z}} c\, dG(\tilde{z})\right]$$

SANITY CHECK: LINEAR CASE

- Back to VF's at the optimal policy

$$\delta'(n^*(z))\underbrace{\left(J(z) - J_0\right)}_{=S(z)} = \pi'(z, n^*(z)) - rc$$

- Given the VFs, this pins down $n^*(z)$ optimal worker's level given z
- Firm *over-hires* compared to a model with no job ladder