External and Internal Market for Managers

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This paper studies the labor market for managers by examining both between and within firm reallocation channels. I document that approximately 40% of the inflows into managerial positions come from internal promotions, a flow comparable in size to the job-to-job transitions into the same roles. I develop a labor search model with internal reallocation and on-the-job learning. External flows depend on how firms are currently internally organized. Internal flows depend on the extent of skill accumulation of workers under managers and on external hiring and separation events. Using administrative data from Germany, I document that managers receive a substantial wage premium and play a key role in the skill development of workers within the establishment. The model matches observed external and internal flows into managerial positions and provides insights into how these channels operate across the talent distribution. Lastly, I evaluate policy implications of targeted Non-Compete Agreements, finding that restrictions on managers lead to fast-tracked promotions and reduced output, whereas non-competes on workers enhance skill accumulation and increase productivity.

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1. Introduction

Finding, training, and retaining managerial talent are often considered crucial drivers of firm productivity. Managers are conceptualized as key employees in supervisory and allocative roles within the firm, where their practices enhance the productivity of the existing internal resources (Bender et al., 2018). With a large knowledge of the production process, managers are also responsible for the diffusion of skills within the firm, making workers under them more productive in current and future tasks (Minni, 2023). This paper studies how firms find and allocate managerial talent by considering two distinct channels: external markets, where firms compete, bargain and poach talent from each other; and internal reallocation, where firms identify and promote talent from within their own ranks.

Using a rich establishment-employee dataset from Germany, we first document that both internal and external channels are substantial in the market for managers. Approximately 40% of inflows into managerial positions come from employees promoted from within the establishment, a figure comparable to the rate of job-to-job transitions into the same positions. This fact highlights the importance of internal reallocation and motivates us to investigate the different implications of these internal and external markets of managerial allocation. In this paper we ask what are the effects of these channels in terms of individual outcomes, including wage growth and the managerial wage premium? From the perspective of firms, how do these channels affect the distribution of managerial skills across firms? And what are the aggregate implications of these forces on productivity?

Addressing these questions is challenging, as macroeconomic models of labor markets are currently not well-equipped to effectively analyze both internal and external reallocation channels. While there is extensive knowledge on external labor flows and their implications for wage ladders and talent allocation, these models often abstract away from firm complexity. Firms are typically represented as either vacancies (Mortensen and Pissarides, 1994; Postel-Vinay and Robin, 2002) or as employers of multiple homogeneous workers in identical roles (Bilal et al., 2022; Elsby and Gottfries, 2022). Conversely, we have a well-established understanding of internal hierarchies and task divisions within complex firms (Garicano and Rossi-Hansberg, 2006; Caliendo et al., 2020). These analyses are usually embedded within a perfectly competitive framework, where there is no role for labor market flows that are crucial in understanding the data. Studying internal and external reallocation channels separately overlooks the interconnected forces that shape labor markets. Internal reallocation is not a isolated event; it is shaped by external market pressures, such as poaching attempts and separations, which influence firm decisions and talent retention. Capturing how these channels interact is crucial to have a complete picture of mobility of managerial talent across firms.

In the first part of this paper we bridge this gap by constructing a labor search model with internal reallocation. We extend the recent work of Herkenhoff, Lise, Menzio, and Phillips (2024) by crucially allowing firms to have an occupational structure: a managerial and non-managerial positions. Managers are necessary for the firm to operate, capturing the notion that the production unit needs an employee in specific supervisory roles so that the other internal resources become productive. Non-managers, or workers, although not necessary for production, when paired with a manager can contribute to output. We allow for asymmetric returns to production coming from the two occupations. Moreover, non-managers learn while paired with a manager, which makes their skill level evolve over time, increasing their productivity in current and future tasks.

Firms operate in a frictional decentralized labor market where they can meet unemployed agents or employees from other firms. Upon meeting, both parties can engage in bilateral bargaining, allowing the firm to attempt a hire when the gains of doing so are larger than the counterpart's outside option. In this setting, external flows depend not only on the productivity of those involved, but also on how are they currently internally allocated. This is because firms can costly choose to reallocate their current employees by promoting a worker to manager or demoting a manager to worker. This implies that it is harder to poach a sitting manager from a firm compared to a non-manager of same skill, as their outside option will take into account the higher value of being allocated as manager at their current firm. A key mechanism in this setting is that firms can make use of the internal skill accumulation of non-managers together with promotions to avoid frictional hiring and keep productivity high despite managerial turnover.

In the second part of this paper, we present novel empirical evidence on managers in the German data. These findings are new and are essential for aligning the model with the data by providing calibration moments. Using detailed 5-digit occupation codes, we observe managerial roles distributed across various positions within firms. This granularity allows us to compare wages between managers and comparable non-managers in terms of occupation and other observable characteristics, such as tenure, age, and education. We find a significant wage premium of 14% for managers, even after controlling for task complexity, which has been identified as a key driver in the German wage ladder (Bayer and Kuhn (2019)). This simple finding is important for two reasons. First, it demonstrates that the label *manager* carries predictive power regarding wage differentials. Second, it aligns well with our theoretical framework, where managerial and non-managerial positions contribute differently to the production function—a distinction that would be reflected in their wages.

We then analyze the flows of managers in the data, distinguishing between internal and external transitions. As noted earlier, inflows into managerial positions are roughly balanced between internal reallocation and external hires. Normalizing by the total number of managers in our sample of establishments, we find that the annual internal inflow into managerial positions is approximately 5.6%, compared to 4.9% from external hires. In contrast, inflows into non-managerial positions follow a more expected pattern in labor markets, with the majority originating from unemployment or job-to-job transitions, and only a small share coming from within the establishment. This finding underscores the importance of analyzing both external and internal inflows in the particular context of managers, as both of these channels play a particularly prominent role for this highly compensated segment of the workforce.

Finally, we provide reduced-form evidence on the manager's role in skill development within the establishment. Focusing on non-managers who transitioned to a different establishment following a spell of unemployment, we find that those who were previously paired with higher-quality managers tend to earn higher wages in their new establishments. Considering an increase of 10% in the average wage of the previous managers, we find a wage increase of 0.7% in the next job, everything else equal. This finding complements previous studies that document the effects of learning in high-quality firms (Gregory, 2020) or from skilled coworkers (Jarosch, Oberfield, and Rossi-Hansberg, 2021; Herkenhoff et al., 2024), by specifically highlighting the role of high-quality managers in workers' skill accumulation.

With these facts in mind, we proceed to calibrate key parameters to target novel moments in the data. The model effectively captures both internal and external flows into management positions, reinforcing our belief that, despite being stylized, the model is able to capture complex mobility patterns that jointly depend on external market forces and internal reallocation decisions. This makes it a suitable framework for analyzing the distribution of managerial talent across firms and its implications for productivity.

Through the lens of the model, we can decompose the external and internal flows into management positions across the distribution of managerial talent. While similar in average magnitude, the calibrated model indicates that external hires are more prevalent at the extremes of the talent distribution: firms either seeking low-skill managers to initiate production, or firms looking for superstars productive managers for their operations. In contrast, internal promotions are more commonly used for mid-level managerial roles, where firms can leverage the employees' accumulated skills by reallocating them into management. Intuitively, few firms retain employees long enough for them to become top-tier managers, so promotions typically occur in the middle of the talent distribution.

Additionally, the calibration exercise provides valuable measures of the production function parameters, specifically the manager's relative share in the output function, which is calibrated to match the managerial wage premium observed in the data. Intuitively, a larger managerial share corresponds to a greater average wage differential between managers and non-managers. The calibrated share of 0.68 highlights the significance of managerial roles within firms, as reflected in the wage differentials seen in the data. In the learning aspect of the model, the calibration indicates a strong learning rate for workers when paired with a manager, exceeding the rates found in Herkenhoff et al. (2024), which focus more broadly on learning from coworkers. This finding highlights the particularly significant role that managers play in the skill accumulation of workers, reinforcing the idea that managerial talent is a key driver of firm productivity by increasing the skills of other employees over time.

Lastly, we explore policy implications of internal and external channels of managerial allocation, using the calibrated model as a laboratory to study Non-Compete Agreements (NCAs). We extend the model to examine cases where firms are restricted from hiring employees under NCAs directly from other firms. Our framework allows us to study targeted NCAs for either managerial or non-managerial roles, as well as the role of internal promotions in scenarios where job-to-job mobility is restricted. In both cases, wages for managers and non-managers decrease, due to the lack of competitive pressure in the external market— the key driver of wage growth in our model.

Interestingly, our results differ when examining aggregate productivity and the skill distribution. When NCAs target managerial roles, in equilibrium firms fast-track promotions, filling managerial positions with employees from the lower-middle range of the talent distribution. This shift reduces skill accumulation in non-managerial roles, hindering learning and lowering the aggregate level of skill and aggregate product of the economy. Conversely, when NCAs apply to non-managers who benefit from on-the-job learning, firms respond by reducing internal promotions across all talent levels. This leads to non-managers staying longer in their roles, enabling skill accumulation, which spillover and raise productivity in managerial positions and overall output. These findings highlight the importance of considering internal reallocation when evaluating labor market policies. When firms can promote talent from within, policies that impact the external market also influence internal incentives, affecting firm composition, the extent of learning within the firm, and ultimately, its overall productivity.

Related literature. The paper is connected to a growing literature that studies frictional labor markets with firm dynamics considerations. These papers relax the assumption on constant returns to scale in production critical to classical papers in search theory (Burdett and Mortensen, 1998; Mortensen and Pissarides, 1994; Postel-Vinay and Robin, 2002). Schaal (2017) examine the role of firm idiosyncratic shocks in a directed search framework with complete contracts, studying their impact on unemployment fluctuations. Similarly, Gouin-Bonenfant (2022) analyze a continuous-time wage-posting model with firm dynamics, where productivity dispersion shields high-productivity firms from wage competition, leading to a low aggregate labor share. Bilal, Engbom, Mongey, and Violante (2022) build a seminal model extend the classical on-the-job search model of Postel-Vinay and Robin (2002) to incorporate firm entry and exit à la Hopenhayn (1992), capturing the dynamic reallocation of workers across firms, interacting with firm size and age. Elsby and Gottfries (2022) develop a tractable continuous-time model where firm dynamics and on-the-job search drive endogenous misallocation through marginal product dispersion. In the context of aggregate shocks, Audoly (2023) introduces state-contingent wage contracts based on Moscarini and Postel-Vinay (2013), which capture firms' responses to macroeconomic fluctuations. Closest to our approach is Herkenhoff, Lise, Menzio, and Phillips (2024), which examines learning from coworkers within a random search model. We extend their framework by introducing an occupational structure within firms, rather than assuming homogeneous or symmetric workers. This addition allows for internal reallocation of resources, adding a new channel of reallocation that is crucial in understanding the data.

Our firm internal occupational is connected to the literature of firm hierarchy and task division from the seminal work of Garicano and Rossi-Hansberg (2006). Caliendo, Monte, and Rossi-Hansberg (2015), Caliendo et al. (2020) build on that to study how firm structure changes is response to shocks and the relationship between internal reorganization of resources and firm growth. Adenbaum (2023) study an assignment model of task-division within firms with endogenous structure. Kohlhepp (2023) examines organization complexity and task assignment to uncover firm specific organization costs that in turn affect endogenous firm structure. All these models consider firm internal organization while abstracting from search frictions that affect the hiring and flow rates in these markets. One notable exception is Freund (2024), who also uses German data to study worker complementarity in a problem with task allocation and external frictional markets. We contribute to his study by focusing on explicit internal reallocation channels and the particular role of managers in skill accumulation.

This paper also contributes to the literature of manager allocation and productivity. The seminal work of Bloom and Van Reenen (2007) measures the impact of managerial practices in firms' productivity and profitability[1]. Minni (2023) focuses on the knowledge of managers in improving the assignment of workers to tasks within the firm, leading to long term gains of productivity. Pastorino (2022) studies internal mobility when firms and workers learn about their underlying ability. Metcalfe, Sollaci, and Syverson (2023) empirically study managers' effect on productivity in the retail sector. Hjort, Malmberg, and Schoellman (2022) study the role of managers in the productivity of firms in developing countries. Friedrich (2023) studies the market for managerial talent in a model with internal promotions where firms take advantage of information frictions and firm specific human capital to retain and promote talent. I contribute to this literature by introducing a setting with internal reallocation in a search model that can rationalize the data on

[1]See Bloom, Lemos, Sadun, Scur, and Van Reenen (2014) for a review

external and internal managerial flows.

There is a vast series of papers that have used the German employer-employee data to study labor market dynamics, including Card, Heining, and Kline (2013), Bender et al. (2018), Bayer and Kuhn (2019), Gulyas (2020), Heise and Porzio (2022). Jarosch, Oberfield, and Rossi-Hansberg (2021) study learning from coworkers in a perfectly competitive setting, while Gregory (2020) measures impacts of firm quality in the skill accumulation of the workers and their life-cycle wage dynamics

Finally, our main counterfactual relates to the literature of non-competes and labor maker outcomes empirically studied by Prescott, Bishara, and Starr (2016) and Johnson, Lavetti, and Lipsitz (2023). Gottfries and Jarosch (2024) develop a model of wage-posting with monopsonistic firms and non-competes. Shi (2023) studies a model of on the job search with expanded contracts to include non-competes. We contribute to this literature by studying the effects of targeted non-competes in a context with firm occupation and internal reallocation.

This paper is organized as follows. Section 2 presents the model, its primitives, and the equilibrium. Section 3 describes the data and presents the empirical evidence on manager flows, wages and their role in skill accumulation, as well as the calibration of the model. Section 4 presents properties of the calibrated model. Section 5 examines counterfactuals with non-compete agreements and Section 6 concludes.

2. Model

In this section we develop a search model with internal reallocation and external hiring, extending the framework of Herkenhoff et al. (2024), by allowing asymmetric production functions, firm occupation structure and skill accumulation that depend crucially on the internal allocation of the firm. In Sections 2.1 and 2.2 we introduce the primitives of the model, its timing and key assumptions. In Section 2.3 we describe the stationary equilibrium of the model.

2.1. Primitives

We consider a labor market composed of a mass N of firms and a mass 1 of agents. Agents are characterized by their skill levels or productivity $z \in \{z_1, ..., z_J\} =: \mathbb{Z}$. There are two distinct roles within each firm: *managers* and *non-managers*. Hereafter non-managers in the text will be simply referred to as *workers*. In particular, a manager is necessary for the firm to operate, while a worker accumulates human capital through on-the-job learning in a process described below.

Firms are constrained to hire at most two agents at any given time: one manager z_m and one worker z_n. Thus, the *state of a firm* is given by the tuple $y = (z_m, z_n)$, where z_m denotes the productivity of the agent employed as a manager and z_n denotes the productivity of the one allocated as worker. Zeros in the firm's state will denote the lack of a manager or worker, respectively. We denote a firm with a manager and a worker as a *team*.

The firm's output depends on the productivity of its manager and worker, as well as their relative shares in the production process. Let the production function of a firm be $f(z_m, z_n)$, which is specified for each possible firm configuration as follows:

$$
\begin{aligned}
f(z_m, z_n) &= z_m^{\alpha} z_n^{1-\alpha}, && \text{with manager and a worker,} \\
f(z_m, 0) &= z_m^{\alpha} z_1^{1-\alpha}, && \text{with only a manager,} \\
f(0, z_n) &= 0, && \text{with only worker,} \\
f(0, 0) &= 0, && \text{firm is vacant.}
\end{aligned}
$$

Here, $\alpha \in (0, 1)$ determines the relative share of the manager's productivity in the firm's output. The intuition behind this specification of the production function is to consider the case where the firm needs an agent performing the managerial role, responsible for supervising and coordinating internal operations. This agent is responsible to make the other resources (which are not explicitly modeled here) productive and ensure the firm is operational. When paired with a manager, the worker contributes to production with a share of $1 - \alpha$. In the absence of a worker, the firm still produces as if it would employ a worker with the lowest possible productivity, z_1. Although workers are not necessary for production in this framework, they represent the firm's internal pool of knowledge. This pool can be internally reallocated into managerial positions without the need for external hires, avoiding the frictions we describe below.

The timing of events in the model inside a period is better understood if divided into four stages: shocks, search & match, reallocation, and production. In the *shocks* stage agents and firms can be affected by a sequence of events that may alter their state for the current period. First, workers z_n that are currently employed together with a manager z_m may experience a learning shock, which updates their productivity following the transition probability $Q(z'_n|z_n, z_m) \geq 0$ with $\sum_{z'_n} Q(z'_n|z_n, z_m) = 1$ for all z_n and z_m. Similarly, unemployed agents may experience shocks to their productivity, represented by $Q_u(z'|z) \geq 0$ with $\sum_{z'} Q_u(z'|z) = 1$ for all z. Finally, agents may permanently exit the model with probability σ. This means that every period an aggregate mass of σ agents exit the labor market and are replaced by a measure σ of newborn agents. The distribution of these entrant agents is given by $\pi(z) \geq 0$ again with $\sum_z \pi(z) = 1$ for all z.

Next, in the *search & match* stage, unemployed agents search for firms, and with probability $\lambda_u \in (0, 1]$ they meet a randomly selected firm. Also, with probability $\lambda \in (0, 1]$ a firm will receive a draw of another randomly selected firm, for which the former could try to poach an agent from the latter. In any type of the meetings above, hiring will occur if and only if the *gains from trade* are positive. For clarity, when a firm is contacted by an unemployed agent it simply computes the gains from trading with that agent and hire with they are greater than zero. When a firm meets another firm, the poaching firm - the one that received the shock- has the opportunity to hire at most one agent employed at the poached firm. The poaching firm will hire the agent that maximizes the gains from trade inside the meeting. If the poaching firm has two agents already, it will have to fire one of them to hire the new agent. The poached firm cannot hire any agents in this stage. We carefully introduce notation and explain the gains from trade for each possible meeting in Section 2.2.

Upon successful poaching, the hiring firm decides the new agent's allocation. If there is an open position, either as a worker or a manager, the new hire can be assigned to that role without cost. However, if the firm chooses to reallocate its current employees, it incurs a cost: $c_p > 0$ for promoting an incumbent worker to manager and $c_d > 0$ for demoting a manager to worker. Finally, agents already employed by a firm may experience exogenous separation with probability $\delta \in (0, 1]$, becoming unemployed. From the point of view of a firm and its currently employed agents, the events of meeting an employed worker, meeting another firm, being poached by another firm or the exogenous separation of an agent are assumed to be all mutually exclusive events inside the search & match stage.

What follows is the *reallocation* stage, during which firms may decide to either separate from one or more employees or reallocate them internally by promoting the worker and/or demoting the manager. These decisions are made based on the gains from trade associated with each option. The costs incurred for promotions and demotions are the same as those in the search & match stage. Specifically, the cost of promoting a worker is denoted by c_p, while the cost of demoting a manager is denoted by c_d. Firms do not incur in any firing costs.

Finally, in the *production* stage, firms produce according to the output function and the state they have reached after the reallocation phase. Firms distribute part of the output as wages to their current employees, while the remainder is retained as profits. Unemployed agents with skill level z engage in home production, generating $b(z) > 0$ units of output.

2.2. Gains from Trade and Wages

In order to explain gains from trade in the model it is useful to introduce some notation. Let $U(z)$ be the continuation value of an unemployed agent of quality z at the production stage. Let $\widetilde{V}(y)$ be the *joint* value of a firm and its employees in state y at the beginning of the search & match stage. Similarly, let $\widehat{V}(y)$ be the joint value at the beginning of the reallocation stage, that followed the search and match and finally $V(y)$ be the joint value at the production stage. The joint values are the sum of the value of the firm and the values of all current employees. We refer to *firm-employees unit* when talking about the joint value of a firm and its employees.

At the search and match stage, inside a meeting, gains from trade are defined as the difference between the *marginal value* to the current firm-employees unit of hiring a new agent and the *outside option* of the new agent at their current firm-employees state.

Decomposing each term, the marginal value of hiring an agent z' to a firm in state y, denoted $\nu_{z'}(y)$, is given by

$$\nu_{z'}(y) = \begin{cases} \max\left\{\widehat{V}(z',0), \widehat{V}(0,z')\right\} - \widehat{V}(0,0) & \text{if } y = (0,0) \\ \max\left\{\widehat{V}(z',0) + U(z_m), \widehat{V}(z_m,z'), \widehat{V}(z',z_m) - c_d\right\} - \widehat{V}(z_m,0) & \text{if } y = (z_m,0) \\ \max\left\{\widehat{V}(0,z') + U(z_n), \widehat{V}(z',z_n), \widehat{V}(z_n,z') - c_p\right\} - \widehat{V}(0,z_n) & \text{if } y = (0,z_n) \\ \max\left\{\widehat{V}(z',z_n) + U(z_m), \widehat{V}(z_m,z') + U(z_n), \widehat{V}(z',z_m) + U(z_n) - c_d, \right. & \\ \qquad\qquad \left. \widehat{V}(z_n,z') + U(z_m) - c_p\right\} - \widehat{V}(z_m,z_n) & \text{if } y = (z_m,z_n) \end{cases}$$

Intuitively, a vacant firm $y = (0,0)$ upon hiring z' will compare the joint value of allocation it as manager $\widehat{V}(z',0)$ or as a worker $\widehat{V}(0,z')$, taking the maximum between them and leaving the state of being a vacant firm $\widehat{V}(0,0)$. A firm with only a manager $y = (z_m,0)$ will compare the joint value of hiring z' as a manager $\widehat{V}(z',0)$ separating from current manager which gives to the firm-employee unit the additional value of $U(z_m)$; the value of allocating the new hire in the empty slot of the worker with total value $\widehat{V}(z_m,z')$; and the value of allocating the new hire as manager, but demoting the incumbent manager with total value $\widehat{V}(z',z_m) - c_d$. A similar reasoning applies to the case of a firm with only a worker $y = (0,z_n)$ with the difference that in the last instance, the firm is promoting the incumbent worker to manager, incurring in the cost c_p. Finally, a firm with a team $y = (z_m,z_n)$ in order to hire z' will have to let one of its employees go. The firm will compare the value of hiring z' as a manager and firing the current manager, against the value of hiring as a worker and firing the current worker and against the options that include promotion or demotion together with the firing of the other employee. All four options are contemplated inside the max operator on the last line of the equation above.

Now the outside option of a *manager* of skill z' currently in state y' is given by

$$u_m(y') = \begin{cases} \widehat{V}(z',0) - \widehat{V}(0,0) & \text{if } y' = (z',0) \\ \widehat{V}(z',z_n) - \widehat{V}(0,z_n') & \text{if } y' = (z',z_n') \end{cases}$$

where for a firm with only a manager $y' = (z',0)$, the outside option when considering losing the manager is the difference between value that the firm-employee unit enjoys under $(z', 0)$ and the value of the vacant firm. For a firm with a team $y' = (z', z_n')$, the outside option is the difference between the value of the firm-employee unit under (z', z_n') and the value of the firm with only the worker, $(0, z_n')$ at the beginning of the reallocation stage. Note that the continuation value $\widehat{V}(0, z_n')$ accounts for the best possible reallocation of the worker, which could include a promotion to manager, as will be explained in detail below.

Analogously the outside option of a *worker* of skill z' in state y' is

$$u_n(y') = \begin{cases} \widehat{V}(0, z') - \widehat{V}(0, 0) & \text{if } y' = (0, z') \\ \widehat{V}(z_m', z') - \widehat{V}(z_m', 0) & \text{if } y' = (z_m', z') \end{cases}$$

which follows the same logic outlined above for the manager's outside option. Finally, the outside option of an *unemployed worker* of skill z' is simply

$$u_{z'} = U(z')$$

the value of being unemployed. The objects $v_{z'}(y)$, $u_m(y')$, $u_n(y')$ and $u_{z'}$ fully describe the gains from trade in any possible meeting in the model, which in turn pins down the transitions of agents across states and the evolution of the firm distributions at the search and match stage. We postpone to Section 2.3 the discussion of the transitions across states that occur during the reallocation stage, as it makes more sense to see it the context of the Bellman equations that constitute the equilibrium of the model.

The notion of gains from trade between and agent and a firm used in this paper is one that includes the effect of the hiring in the continuation value of the joint firm-employee unit, from both the poaching and poached firms. The view of these assumptions is that all members of the production unit are able to use either implicit or explicit contractual clauses to make sure the individuals fully internalize the effect of any employment decision into each other, and hence only care about maximizing the joint value of the firm-employee unit. This is in line with recent multi-worker search models as in Herkenhoff et al. (2024) and follow the common intuition in the labor search literature where firm-employee pairs remain together if and only if the total value of the match is positive, as in Mortensen and Pissarides (1994), Postel-Vinay and Robin (2002). More robustly, Bilal et al. (2022) develop an explicit bargain procedure within the production unit with multiple homogenous employees that leads to same notion of joint gains that we adopt in this paper.

With the transitions pinned down by the gains from trade inside a meeting we can turn attention to wage determination. Upon hiring, we assume the division of the gains from trade is determined via bargaining, where the hired agent captures a share γ of the gains from trade, while the firm retains the remaining share $1 - \gamma$.

The employee's share is delivered through a wage, which remains constant unless two events arise. First, if the employee's outside option exceeds the current marginal value of the firm-employee unit, the wage must increase. This happens either because the employee is poached by another firm, when the gains from trade are positive, or because the incumbent firm is able to retain the employee after a poaching attempt, when the gains from trade are negative but the poaching firm's offer exceeds the employee's current value. This ensures that the employee always has a private incentive to remain with the firm in which the gains from trade are positive.

Second, if the employee's value falls below the actual marginal value to the firm-employee unit, the wage decreases. This can occur after shocks or separations that reduce the marginal value of the employee to the production unit. This guarantees that if offers were bilateral and private information, the employee would always have an incentive to reveal all outside offers to the firm, to avoid wage cuts.

The assumption that wages remain constant over time unless there is a change in the employee's outside option or the marginal value of the firm-employee unit is natural in a search setting (Cahuc, Postel-Vinay, and Robin, 2006). Due to frictions, there is no constant competitive pressure on wages that would push them to reflect the current marginal value of the employee. Instead, wages only adjust when these infrequent pressures arise, prompting the employee to make the correct allocative decisions.

In our setting, promotions are not necessarily priced in the wage, but rather in the agent's continuation value. A wage increase following a promotion only occurs if, in the same period, the firm faces a poaching attempt, and the outside offer forces the firm to adjust wages. This happens when the firm correctly anticipates the value of promoting the worker during the reallocation stage. Consequently, the model allows for multiple promotion events that do not coincide with immediate wage increases. However, the employee's continuation value still rises with promotion due to the higher expected future wages resulting from future labor market flows. A newly promoted manager becomes more valuable to the firm, meaning that when approached by a poaching firm, the incumbent will value the promoted employee more highly than before.

An alternative approach would be to specify an internal bargaining game within the firm, where the gains from promoting the worker would be explicitly considered and split. The overall structure of the model, particularly the equilibrium conditions in Section 2.3, would remain intact. However, this would require making assumptions about how the firm and worker negotiate to share the surplus at the time of promotion. Instead, we take a more agnostic approach, consistent with search models, where in the absence of external pressures, there are no incentives to adjust the current wage.

2.3. Equilibrium

In what follows we describe the Bellman equations that constitute the stationary equilibrium conditions of the model. It is convenient to follow the values of firm-employee units at each stage inside a period. To this end we introduce notation for distributions of agents and firms across states. Let the mass of unemployed agents with productivity z be denoted by $e_u(z)$, while the mass of firms in a particular state (z_m, z_n) is represented by $e(z_m, z_n)$. The distributions of firms across different states, as well as the unemployment distribution of workers, are key endogenous objects in the model, determined by the dynamics of hiring, firing, and on-the-job learning. At any point in time we must have

$$\sum_y e(y) = N \quad \text{and} \quad \sum_z e_u(z) + \sum_{z_m} e(z_m, 0) + \sum_{z_n} e(0, z_n) + 2 \sum_{z_m, z_n} e(z_m, z_n) = 1$$

reflecting the total mass of firms N and the total mass of agents 1, which accounts for the fact that firms with a team count as two agents.

At the production stage, the continuation value of an unemployed agent of skill z is given by

$$U(z) = b(z) + \beta \mathbb{E}_{z'} \left[U(z'_m) + \sigma[0 - U(z')] + (1 - \sigma) \left[\sum_y \frac{\lambda_u e(y)}{N} \gamma \left[\nu_{z'}(y) - U(z') \right]^+ \right] \right] \tag{1}$$

where after receiving the home production $b(z_m)$ the unemployed agent enter the next period with a new draw of skill z' and faces the probability of exiting the model with probability σ. The second term captures the expected value of meeting a firm at the search and match stage, where with probability λ_u the agent receives a draw and with probability $e(y)/N$ meet each type of firm at state y. The notation $[x]^+$ denotes the $\max\{x, 0\}$ operator. If the gains from trade, term within brackets, are positive, the agent will be hired capturing a share γ of it.

For firm-employee units the values at the beginning production are given by

$$V(0,0) = f(0,0) + \beta \widetilde{V}(0,0)$$
$$V(z_m, 0) = f(z_m, 0) + \beta \left[\sigma \widetilde{V}(0,0) + (1 - \sigma) \widetilde{V}(z_m, 0) \right]$$

$$V(0, z_n) = f(0, z_n) + \beta \left[\sigma \widetilde{V}(0,0) + (1 - \sigma) \widetilde{V}(0, z_n) \right] \tag{2}$$

$$V(z_m, z_n) = f(z_m, z_n) + \beta \left[\sum_{z'_n} Q(z'_n | z_n, z_m) \left(\sigma^2 \widetilde{V}(0,0) + \right. \right.$$

$$\sigma(1-\sigma)\left(\widetilde{V}(z_m,0)+\widetilde{V}(0,z_n')\right)+(1-\sigma)^2\widetilde{V}(z_m,z_n')\Big)\Big]$$

where we have the value of a vacant firm, followed by the joint value of a firm with a manager only, a worker only, and finally a firm with a team. Firm produce at their current state follow the production function f. In the next period, units with employees face the probability of such agents exiting the model with probability σ. Additionally, firms with a team can experience learning shocks of the worker's productivity, which are captured by the transition probabilities $Q(z_n'|z_n, z_m)$.

Next, we describe the values for firm-employees units at the beginning of the search and match stage. These are somewhat cumbersome to writer as they need to take into account all possible types of meeting that the units can have. The value for a vacant firm is given by

$$\begin{aligned}
\widetilde{V}(0,0) &= \widehat{V}(0,0) \\
&+ \sum_{z'} \frac{\lambda_u e_u(z')}{N}(1-\gamma)\left[\nu_{z'}(0,0)-u_{z'}\right]^+ \\
&+ \sum_{z_m'} \frac{\lambda e(z_m',0)}{N}(1-\gamma)\left[\nu_{z_m'}(0,0)-u_m(z_m',0)\right]^+ \\
&+ \sum_{z_n'} \frac{\lambda e(0,z_n')}{N}(1-\gamma)\left[\nu_{z_n'}(0,0)-u_n(0,z_n')\right]^+ \\
&+ \sum_{y_t'} \frac{\lambda e(y_t')}{N}(1-\gamma)\left[\max\left\{\nu_{z_m'}(0,0)-u_m(y_t'), \nu_{z_n'}(0,0)-u_n(y_t')\right\}\right]^+
\end{aligned} \tag{3}$$

In each line of equation (3) we have the continuation value of the vacant firm in each possible meeting type. The first line simply describes the case where nothing happens at search and match stage, and the firm enter the reallocation stage still vacant. The second line describes the case where the vacant firm meets an unemployed agent, where a mass $\lambda_u e_u(z')$ receive the meeting shock and meet the firm with probability $1/N$, for every $z' \in \mathcal{Z}$. If the gains from trade are positive, the firm will hire the agent, capturing a share $(1-\gamma)$. Similarly, the third and fourth lines describe the case where the vacant firm meets a firm with a manager z_m' or a worker z_n', respectively which happens with probability $e(z_m', 0)/N$ and $e(0, z_n')/N$ conditional on the received meeting shock λ. Finally, the last line describes the case where the vacant firm meets a team of two agents at state $y_t' = (z_m', z_n')$, where the firm will hire the agent that maximizes the gains from trade, which is the maximum between the two options of poaching the manager or the worker, conditional on the gains from trade being positive.

The joint value for firm-employees units with only a manager is given by

$$
\begin{aligned}
\widetilde{V}(z_m, 0) = \widehat{V}(z_m, 0) &+ \delta\left[\widehat{V}(0,0) + U(z_m) - \widehat{V}(z_m, 0)\right] \\
&+ \sum_{z'} \frac{\lambda_u e_u(z')}{N}(1-\gamma)\left[\nu_{z'}(z_m, 0) - u_{z'}\right]^+ \\
&+ \sum_{z'_m} \frac{\lambda e(z'_m, 0)}{N}(1-\gamma)\left[\nu_{z'_m}(z_m, 0) - u_m(z'_m, 0)\right]^+ \\
&+ \sum_{z'_n} \frac{\lambda e(0, z'_n)}{N}(1-\gamma)\left[\nu_{z'_n}(z_m, 0) - u_n(0, z'_n)\right]^+ \\
&+ \sum_{y'_t} \frac{\lambda e(y'_t)}{N}(1-\gamma)\left[\max\left\{\nu_{z'_m}(z_m, 0) - u_m(y'_t), \nu_{z'_n}(z_m, 0) - u_n(y'_t)\right\}\right]^+ \\
&+ \sum_{y} \frac{\lambda e(y)}{N}\gamma\left[\nu_z(y) - u_m(z_m, 0)\right]^+
\end{aligned}
\tag{4}
$$

and with only a worker

$$
\begin{aligned}
\widetilde{V}(0, z_n) = \widehat{V}(0, z_n) &+ \delta\left[\widehat{V}(0,0) + U(z_n) - \widehat{V}(0, z_n)\right] \\
&+ \sum_{z'} \frac{\lambda_u e_u(z')}{N}(1-\gamma)\left[\nu_{z'}(0, z_n) - u_{z'}\right]^+ \\
&+ \sum_{z'_m} \frac{\lambda e(z'_m, 0)}{N}(1-\gamma)\left[\nu_{z'_m}(0, z_n) - u_m(z'_m, 0)\right]^+ \\
&+ \sum_{z'_n} \frac{\lambda e(0, z'_n)}{N}(1-\gamma)\left[\nu_{z'_n}(0, z_n) - u_n(0, z'_n)\right]^+ \\
&+ \sum_{y'_t} \frac{\lambda e(y'_t)}{N}(1-\gamma)\left[\max\left\{\nu_{z'_m}(0, z_n) - u_m(y'_t), \nu_{z'_n}(0, z_n) - u_n(y'_t)\right\}\right]^+ \\
&+ \sum_{y} \frac{\lambda e(y)}{N}\gamma\left[\nu_{z_n}(y) - u_n(0, z_n)\right]^+
\end{aligned}
\tag{5}
$$

where the logic for equations (4) and (5) follows the same overall structure as in equation (3). The differences are that we must take into account the possibility of outflows from the firm-employee unit that either happens exogenously following the shock with probability δ or through poaching events from other firms. The latter is presented in the last line of both equations, where we consider that for any other firm $y \in \mathcal{Y}$ a mass $\lambda e(y)$ that received the meeting shock will have the possibility of meeting the firm-employee unit that happens with probability $1/N$. If the gains from trade are positive, the employee is hired, capture γ of it, which accrues to the firm-employee unit.

The exact same structure can be found in equation (6) for the joint value of a firm with a team of two employees, which we write below for convenience.

$$
\begin{aligned}
\widetilde{V}(z_m, z_n) = \widehat{V}(z_m, z_n) \\
&+ \delta\left[\widehat{V}(0, z_n) + U(z_m) - \widehat{V}(z_m, z_n)\right] + \delta\left[\widehat{V}(z_m, 0) + U(z_n) - \widehat{V}(z_m, z_n)\right] \\
&+ \sum_{z'} \frac{\lambda_u e_u(z')}{N}(1-\gamma)\left[\nu_{z'}(z_m, z_n) - u_{z'}\right]^+ \\
&+ \sum_{z'_m} \frac{\lambda e(z'_m, 0)}{N}(1-\gamma)\left[\nu_{z'_m}(z_m, z_n) - u_m(z'_m, 0)\right]^+ \\
&+ \sum_{z'_n} \frac{\lambda e(0, z'_n)}{N}(1-\gamma)\left[\nu_{z'_n}(z_m, z_n) - u_n(0, z'_n)\right]^+ \\
&+ \sum_{y'_t} \frac{\lambda e(y'_t)}{N}(1-\gamma)\left[\max\left\{\nu_{z'_m}(z_m, z_n) - u_m(y'_t), \nu_{z'_n}(z_m, z_n) - u_n(y'_t)\right\}\right]^+ \\
&+ \sum_{y} \frac{\lambda e(y)}{N}\gamma\left[\max\left\{\nu_{z_m}(y) - u_m(z_m, z_n), \nu_{z_n}(y) - u_n(z_m, z_n)\right\}\right]^+
\end{aligned}
\tag{6}
$$

After the events in the search and match stage, firms can reallocate current employees, which include promotions, demotions, and separations. The firm-employees units look for the option that maximize the joint value in the subsequent production stage. Given production values V and unemployment value U, the units solves

$$
\begin{aligned}
\widehat{V}(0,0) &= V(0,0) \\
\widehat{V}(z_m, 0) &= \max\left\{V(z_m, 0), V(0,0) + U(z_m), V(0, z_m) - c_d\right\} \\
\widehat{V}(0, z_n) &= \max\left\{V(0, z_n), V(0,0) + U(z_n), V(z_n, 0) - c_p\right\} \\
\widehat{V}(z_m, z_n) &= \max\{V(z_m, z_n), V(z_m, 0) + U(z_n), V(0, z_n) + U(z_m), \\
&\qquad V(0, z_m) + U(z_n) - c_d, V(z_n, 0) + U(z_m) - c_p, \\
&\qquad V(z_n, z_m) - c_d - c_p, V(0,0) + U(z_m) + U(z_n)\}
\end{aligned}
\tag{7}
$$

where in (7) we have the values for the vacant firm, a firm with only a manager, a firm with only a worker, and firms with a team respectively. Vacant firms simply stay idle. Firm with a manager only choose between staying idle, separating from the manger which produces the joint value $V(0,0) + U(z_m)$, or demoting the manager with cost c_d. The same logic applies to the worker only firm, where the firm can choose to stay idle, separate from the worker, or promote the worker to manager with cost c_p. Finally, the firm with a team chooses between seven options, which include staying idle, separating from either employee without reallocation, promoting or demoting together with separation of the other employee, reallocation both employees or separating from both. Each of these options are represented in the max operator, which captures the best possible outcome for the firm-employee unit.

We are in position to define a stationary equilibrium.

DEFINITION 1. *A stationary equilibrium is such that the values* $\{U(z), V(y), \widehat{V}(y)\}$ *satisfy the Belman equations (1)-(7) taking as given the distributions* $\{e_u(z), e(y)\}$ *for every skill level* $z \in \mathcal{Z}$ *and for every state* $y \in \mathcal{Y}$.

In turn, the distributions $\{e_u(z), e(y)\}$ *are stationary given the policies functions implied by the optimal value functions.*

The hiring, allocation and reallocation policy functions can be found in the Model Appendix A, as they are straightforward given the equilibrium values. We also refer to the model appendix for the definition and derivation of the stationary distribution, which, while intuitive, are rather cumbersome to derive from the policy functions. The value functions for the workers and the equilibrium conditions for the wages can be found in the Appendix B. Wages are inverted from the equilibrium values, and although they are not necessary to characterize the equilibrium, they will be key in the model simulations and connecting data and model generated moments.

3. Data and Empirical Evidence

This section uses information on establishment-employee matches and wages from the longitudinal version of the German Linked-Employer-Employee dataset (LIAB) to document a set of novel facts on managers in terms of wages, flows, and their role on worker's learning within the establishment. After a brief descrption of the data, each section will provide us moments that will be directly used in bringing the model to the data.

3.1. Data Description - LIAB

The LIAB-7519 dataset is constructed from a representative sample of German establishments, capturing the entire workforce from 2008 to 2017. Additionally, it provides access to the complete labor market biographies of workers from these establishments, spanning from 1975 to 2019. This data includes information on wages, occupations, and firm characteristics such as industry, region, and firm size. Following the approach in Dauth and Eppelsheimer (2020), we organize the data into a yearly panel. Further details on sample selection are provided in the Data Appendix C.

A key feature of our study is the use of granular occupation codes to identify managers. By accessing the 5-digit occupation codes (*KLdB 2010*), we can decompose each occupation into three dimensions (Paulus, Matthes et al., 2013). The first three digits represent the occupation group, providing an overall yet detailed classification of the position. The fourth digit allows us to distinguish managers from non-managers within each occupation group, offering a hierarchical perspective on supervisory or executive roles. The final digit captures task complexity, indicating the qualification level required for the position. Throughout the analysis, we designate managers using this occupation code breakdown, allowing us to identify managerial roles not only at the top level (e.g., CEOs) but also across various occupations within firms. Consistent with the language of the model, we will refer to non-managers as workers. For data quality purposes on the manager labeling we further restrict the period of analysis over the sample of establishment to 2013-2017. In the subsequent sections, we leverage the ability to compare managers and workers within the same occupation and with similar task complexity.

3.2. Wages and Manager Premium

We begin by examining the cross-sectional differences in wages across manager and workers. Managerial positions comprise about 10% of the workforce in the establishment in our sample, while commanding around 15% of the wage bill. In order to refine this implied premium and control for other characteristics that affect wages we run the following regression.

$$\text{lwage}_{it} = \beta_0 + \beta_1 \text{Manager}_{it} + \Gamma X_{it} + \epsilon_{it} \tag{8}$$

where lwage_{it} is the log wage of individual i at time t, Manager_{it} is a dummy variable that takes the value of 1 if the individual is a manager at time t, and X_{it} is a vector of individual and firm characteristics. The controls include dummies for task complexity, age, tenure, education, firm size. We take fixed effects of year, 3-gigit occupation, industry, and region. We present the results for three specifications in Table 1. The parameter of interest is β_1, which captures the wage premium of managers relative to workers, controlling crucially for agents in the same occupation and with similar task complexity.

Specifications (1) and (2) show that managers earn approximately 60% more than workers when only year fixed effects and individual characteristics are considered, aligning with the raw data on wage bills. Including firm characteristics and task complexity in specification (3) reduces the manager premium to around 14%. Task complexity is highly correlated with higher wages, consistent with findings in the German context (Bayer and Kuhn, 2019). Nevertheless, the manager differential remains significant and substantial: all else equal, managers earn, on average, 14% more than workers within the same occupation and with similar task complexity.

The interpretation is not causal, of course, as selection into manager positions depends

TABLE 1. Cross-sections Wage Regressions

	(1)	(2)	(3)
Manager	0.672	0.609	0.141
	(0.00)	(0.00)	(0.00)
Complexity level 2			0.184
			(0.00)
Complexity level 3			0.444
			(0.00)
Complexity level 4			0.587
			(0.00)
Year FE	✓	✓	✓
Individual Demographics		✓	✓
Firm Characteristics			✓
R^2	0.051	0.155	0.586
N	2928168	2928168	2183213

Notes: Standard errors are clustered at the establishment level for all the specifications. P-values in parentheses. Controls include dummies for task complexity, age, tenure, education, firm size. Fixed effects of year, 3-digit occupation,2digit industry, and region.

on unobserved characteristics, such as ability, that are correlated with wages. However, the results provide a suggestive evidence that the label manager carries predictive power in terms of higher wages. Moreover, these findings will be used to calibrate the model, helping to identify the managerial contribution to output within our framework.

3.3. Manager Flows

In this section we decompose the internal and external flows into managerial positions. We use our representative sample establishment to keep track of movements of managers and workers inside and outside the establishment. The complete biographies allow us to keep track of the occupations of the agents prior to joining the sample. We present the decomposition of flows in Table 2. In the first column we see the flows into manager, from all the possible origins in our decomposition: manager from the same establishment, manager from a different establishment, worker from the same establishment, worker from a different establishment, and unemployment. These are computed as annual flows as percentage the total number of managers of that year in our sample. Same firm flows restrict agent that in consecutive years are employed in by the same employer ID. Manager from the same establishment changed occupations but preserving managerial status, while workers from the same establishment are those that got promoted to manager from one year to the next. The same logic applies to the external flows with agents that are employed in different establishments.

The largest single inflow into managerial positions comes from workers promoted within the same establishment, accounting for 4.7% of all managers and approximately 36% of total inflows into managerial roles. When combined with inflows from existing managers within the same establishment, internal reallocation comprise about 43% of all managerial inflows. External job-to-job transitions contribute a similar proportion, with 2.1% of managers coming from other establishments where they were already managers and 2.8% from workers moving from different establishments.

For comparison, the second column of Table 2 shows inflows into worker positions as a percentage of the total workforce in these roles. The largest inflow comes from job-to-job transitions across establishments, accounting for 7.10% of all workers. Internal reallocation among workers is relatively smaller compared to managers, with only 2.1% of agents changing roles within the same establishment. A substantial inflow of workers also comes from unemployment of 7.5%, larger than the same rate for managers. These differences motivate a closer examination of internal markets specifically for managers, where internal reallocation plays a more prominent role. Focusing solely on external markets—whether through unemployment or transitions from other establishments—may not fully capture the dynamics of managerial positions as well as it does for more general non-managerial occupations.

3.4. Worker Learning with Managers

Our last set of moments focuses on the role of managers in the diffusion of knowledge and learning within the establishment. We want to assess if the presence of a more skilled manager can affect the stock of knowledge that the worker carries to future tasks. In the spirit of Jarosch, Oberfield, and Rossi-Hansberg (2021) and Herkenhoff et al. (2024),we employ a reduced-form strategy to measure learning from managers. We restrict the sample to workers who, in a given year t, were employed at an establishment with at least one manager, experienced a job-loss episode in $t+1$, and subsequently regained employment at another establishment by the reference date.

The focus on transitions through unemployment spells aligns with the model mechanisms: when the agent is hired by a new establishment, he must bargain with the new firm over the terms of trade and, hence, his wage must reflect only the stock of skills that he has accumulated in past jobs. If we were considering individuals that remaining in the same establishment or transitioned job-to-job without an unemployment spell, any changes in the wage would be contaminated by other factors affecting the marginal value

TABLE 2. Decomposition of Manager Flows

Origin	Into Manager	Into Worker
Manager		
Same Establishment	0.90%	0.17%
Different Establishment	2.10%	0.17%
Worker		
Same Establishment	4.70%	2.10%
Different Establishment	2.80%	7.10%
Unemployment	2.40%	7.50%

Notes: Average annual flows constructed from the representative sample of establishments for which we can observe the entire workforce. Flows into manager are computed as percentage of the total number of managers in that year, and flows into worker are computed as percentage of the total number of workers in that year.

of the original firm, being an inaccurate measure of learning. With this in mind we run the following regression:

$$\text{lwage}_{i,t+1} = \theta_0 + \theta_1 \text{lwage}_{i,t} + \theta_2 \text{ M Wage}_{i,t} + \Theta X_{it} + \epsilon_{i,t} \tag{9}$$

where $\text{lwage}_{i,t+1}$ is the log wage of individual i at time $t+1$ -after the job-loss episode-, $\text{lwage}_{i,t}$ is the log wage of individual i at time t at the original establishment. M $\text{Wage}_{i,t}$ captures our measure of manager knowledge, for which we will explore several specifications described below. X_{it} is a vector of individual and firm characteristics. The controls include again task complexity, age, tenure, education, firm size; with fixed effects of year, 3-digit occupation, industry, and region. Importantly we are also controlling for the average wage of the firm, excluding the individual i and the manager wage. This is an effort to separate the effect of the manager from the overall quality of the firm.

Table 3 presents the results for three specifications. The parameter of interest, θ_2, captures the extent to which the manager's knowledge influences the worker's wage following the job-loss episode. In the first specification, shown in column (1), we use the average wage of all managers in the establishment as a measure of managerial quality. In column (2), we narrow the scope further by using the average wage of managers in the same 3-digit occupation as the worker, following the intuition of Jarosch, Oberfield, and Rossi-Hansberg (2021) that a more granular definition of teams within establishments better captures learning effects. Intuitively, sharing the same occupation suggests that the manager and worker likely work closely together, potentially making the manager's

TABLE 3. Worker Learning from Managers within Establishments

	(1) Avg Manager	(2) Same Occup.	(3) Top Manager Same Occup.
θ_1	0.361	0.330	0.329
	(0.00)	(0.00)	(0.00)
θ_2	0.080	0.071	0.320
	(0.02)	(0.03)	(0.01)
N	10655	4104	4104

Notes: Standard errors are clustered at the establishment level for all the specifications. P-values in parentheses. Controls include dummies for task complexity, age, tenure, education, firm size, firm average wage. Fixed effects of year, 3-digit occupation, 2-digit industry, and region.

knowledge more relevant to the worker. Finally, in column (3), we restrict the measure further to the wage of the top manager within the worker's 3-digit occupation, offering a sharper comparison by examining the impact of the most skilled manager on the worker.

The results indicate that the measures of manager quality have a positive and significant effect on the worker's wage at their next job. The coefficients for specifications (1) and (2) are similar, suggesting that a 10% increase in the average wage of managers corresponds to a 0.8% increase in the worker's wage, all else equal. The estimate for specification (3) is larger, with a 10% increase in the top manager's wage associated with a 3.2% increase in the worker's wage, indicating that the top manager within the same occupation plays a particularly influential role in the worker's skill accumulation.

The estimates for θ_1 are also positive and significant, remaining consistent across specifications. This parameter intuitively captures the worker's accumulated knowledge, reflected in their previous wage, and how much of this skill transfers to the new job. Controlling for θ_1 allows us to isolate any learning from the manager that was not fully reflected in the worker's wage at the previous job.

The interpretation of these parameters is not structural, as wages are noisy indicators of an agent's skill, and the allocation patterns of workers and managers across establishments are not random. However, these regressions will play a crucial role in guiding the learning process within our model. In Section 3.5, we use the model to translate these coefficients into parameters for our learning process.

3.5. Calibration

We jointly calibrate seven key moments to target eight moments in the data. The calibration is done at monthly frequency, and we focus on the flows of managers, the wage premium, and the learning parameters presented in the previous sections. The parameters, their description and values are presented in table 4. Although the model is jointly calibrated there is a clear mapping between each of the parameters and a particular empirical moment.

The costs of promotion c_p and demotion c_d are adjusted to target the internal rates of promotion and demotion respectively. Intuitively, controlling how costly it is to reallocate an agent within the firm will affect how often each type of firm will do so. We obtain, in bot cases a cost of around 18% of the average firm output, which is substantial, implying a steep cost for low productivity firm-employee units to promote or demote their agents. It is interesting to note that although the calibrated values are similar, the target moments are not, with the promotion rates being four times larger than the demotion rates. This is likely due to the nature of the model where given the limited slots in the firm, demotions would be much more common here than in more flexible firm structures in the data.

For the share of the manager in the production function α we're targeting the cross-sectional wage premium in the regression (8), by running the analogous specification in the model. The more productive is the manager compared to the worker, the larger the wage premium will tend to be, even when taking into account the search frictions effect on wage described in the model section 2.2. The calibrated value of 0.683 implies that the manager is responsible for a substantial share of the output, consistent with a significant wage premium that we see in that data, even when controlling for a range of observable characteristics.

For the learning process we parameterize as follows. When paired with a manager in a team, the worker in z_j moves up one spot in the skill ladder $\mathcal{Z}$ with fixed probability

$$Q(z_{j+1}|z_j, z_m) = Q_{up}$$

for every quality of the manager z_m and stay in the same level with the remaining probability. Naturally, if the worker's skill is the highest level of the skill ladder, the worker stays in that level with probability 1. We calibrate the learning parameter Q_{up} to target the coefficients of the learning regression (9), both t_1 and θ_2. Intuitively, in the model, a worker paired with a manager accumulates skill which might be partially captured in current wages. This worker when transitioning through unemployment will do so with a higher skill that will be reflected as a better future wage when hired again from the unemployment pool. Controlling for skill, being paired with a better manager results in a relatively more *stable* firm-employee unit, which allows the worker to accumulate

TABLE 4. Calibration Parameters

Parameters	Description	Targets	Values
c_p	Promotion Cost (% avg prod.)	Promotion rate	18.39%
c_d	Demotion Cost (% avg prod.)	Demotion rate	18.68%
α	Manager Share	M. wage premium	0.683
Q_{up}	W Learning w/ M	θ_1, θ_2 Learning reg.	0.047
λ	Meeting rate	External M hire rate	0.034
λ_u	Unemp. Meeting rate	Unemp. to M hire rate	0.058
δ	Separation rate	M to Unemp. rate	0.001
Pre-set	**Description**	**Source**	**Values**
χ	Dispersion of newborns	Herkenhoff et al. (2024)	2.62
Q_u^-	Unemp. Skill Depreciation	Herkenhoff et al. (2024)	0.016
b	Flow value of unemployment	Freund (2024)	0.71
β	Discount Factor	10% annual rate	0.992
σ	Exit probability	40 years in the mkt.	0.0021
γ	Employee share of gains	Standard	0.5

skill for longer, again resulting in a higher wage when transitioning to a new firm. The calibrated value of 0.047 is higher than in Herkenhoff et al. (2024), which is due to the focus on managerial markets, where agents less likely to transition to unemployment, and thus have more time to accumulate skills.

For the meeting probabilities λ, λ_u and the exogenous separation probability δ calibration is more standard to target the flows job to job, unemployment to job and job to unemployment respectively. But here, we focus on the inflows into managerial positions that were presented in the previous section, given the focus and nature of the model on labor market for managers. We can see that the values associated to these parameters are substantial smaller than compared to similar models (Freund (2024), Herkenhoff et al. (2024)). The reason for that is the focus on managerial flows that are relatively smaller than for broader flows. In particular the flows from and to unemployment are much smaller for managerial positions, which makes the meeting and separation probabilities smaller in this context.

We pre-set the remaining six parameters of the model, using values from the literature. From Herkenhoff et al. (2024) we take the values for the dispersion of newly entered agents in the economy after an exit event χ and the probability of skill depreciation of unemployed agents Q_u^-. The flow value of unemployment b is taken from Freund (2024), where following the literature it used as the share of output that the agent can get from home production, compared to producing alone at a firm. Finally, the discount factor β is set to a standard value of 0.992 which corresponds to a 10% annual rate, in line with the search literature[2]. The exit probability is set to match an average duration of 40 years in the market, and the employee share of gains is set to a standard value of 0.5.

The model does a reasonably good job at targeting the proposed moments, as it is presented on table 5. In particular, it is able to capture simultaneously the internal promotion rates and external job-to-job hiring rates of managers, implying that the setting is able to reproduce realistically the complex pattern of inflows into managerial positions. By extending existing models to accurately reflect both internal and external flows, our model advances our understanding of managerial mobility and offers a valuable tool for analyzing labor market dynamics taking into account substantial reallocation that occurs within the firm.

Our calibration exercise also manages to capture well the moments related to the wage premium of managers, as well as the learning parameters that are reflected in the worker's wage after transitioning to a new establishment. On the simulated data we run the analogous regressions to (8) and (9) to target such moments. With this we effectively have used the model to transform regression parameters that otherwise have no structural interpretation into targets to discipline the output function, as in the share of the manager in production, and the learning process of workers that occur when paired with managers.

As mentioned above, the area where the model struggles is in the calibration of rates to and from unemployment, which are substantially higher in the model than in the data, even with substantially lower meeting rates form unemployment and separation rate. This is likely due to the nature of the model, where the simplification of two slots implies that whenever a team hires a new worker, one of the agents must be separated, which prompts a higher turnover than observed in the data. Likewise, when hiring from unemployment, vacant firms will likely directly allocate the agent as manager, which inflates the UE rates to managers in the model. More work on this front is needed to improve the calibration of these moments.

4. Properties of the Model

In this section we present a set of properties of the model to illustrate the mechanisms at play. We begin by plotting some policy functions from the calibrated model, which will help us understand the hiring and allocation decisions of firms.

[2]As in Postel-Vinay and Robin (2002),we assume agents' utility to be linear in wages for tractability, and load the role of risk-aversion on the discount factor.

TABLE 5. Data and Model Moments.

Moments	**Data**	**Model**
Promotion rate	0.47%	0.51%
Demotion rate	0.11%	0.25%
External M hire rate	0.41%	0.56%
Unemp. to M hire rate	0.20%	0.87%
M to Unemp. rate	0.07%	0.73%
M. wage premium	0.14	0.15
θ_1 Learning	0.33	0.37
θ_2 Learning	0.07	0.07

4.1. Policy Functions

Figure 1 illustrates the hiring and allocation policies of a firm with only a manager $(z_m, 0)$ across various potential meetings. In each subplot, the vertical axis represents the type of the poaching firm, including vacant firms, and each for level of z_m. The horizontal axis indicates the type of the potential hire, with each subplot corresponding to a different state for this potential hire.

Row 1A shows us if the firm hires when meeting an unemployed agent or a firm with a single employee. Conditional on hiring, we have the allocation of the new employee in $(z_m, 0)$. We can see that when meeting an unemployed agent, every such firm will hire. This reflects the relatively low outside option for the unemployed agents. Vacant firms allocate all possible hires directly into manager positions, which is not an obvious outcome. Given the reallocation costs and learning, the firm could have opted to allocate the new hire as a worker if the types were low, and wait for a better manger to fill the spot. In our quantification however, this force is outweighed by the direct gains of becoming a productive firm. On the same subplot, we can see a threshold region that describes whether the agent is hired as a worker or as manager. If the skill level of the unemployed agent is low enough relative to the incumbent manager, the firm allocates the new hire as a worker. Conversely, the new hire is allocated as manager, demoting the current one. This threshold is not on the 45 degree line because of the costs of demoting the sitting manager, which creates some firms with less productive managers than workers. Moving along row 1A we see similar patterns for hiring from single employee firms, with two exceptions. Vacant firms cannot poach from firms with a manager- there are no gains from trade from leaving the poached firm vacant- but do poach from worker only firms, because they can allocate the hire as manager saving the cost that the poached firm would

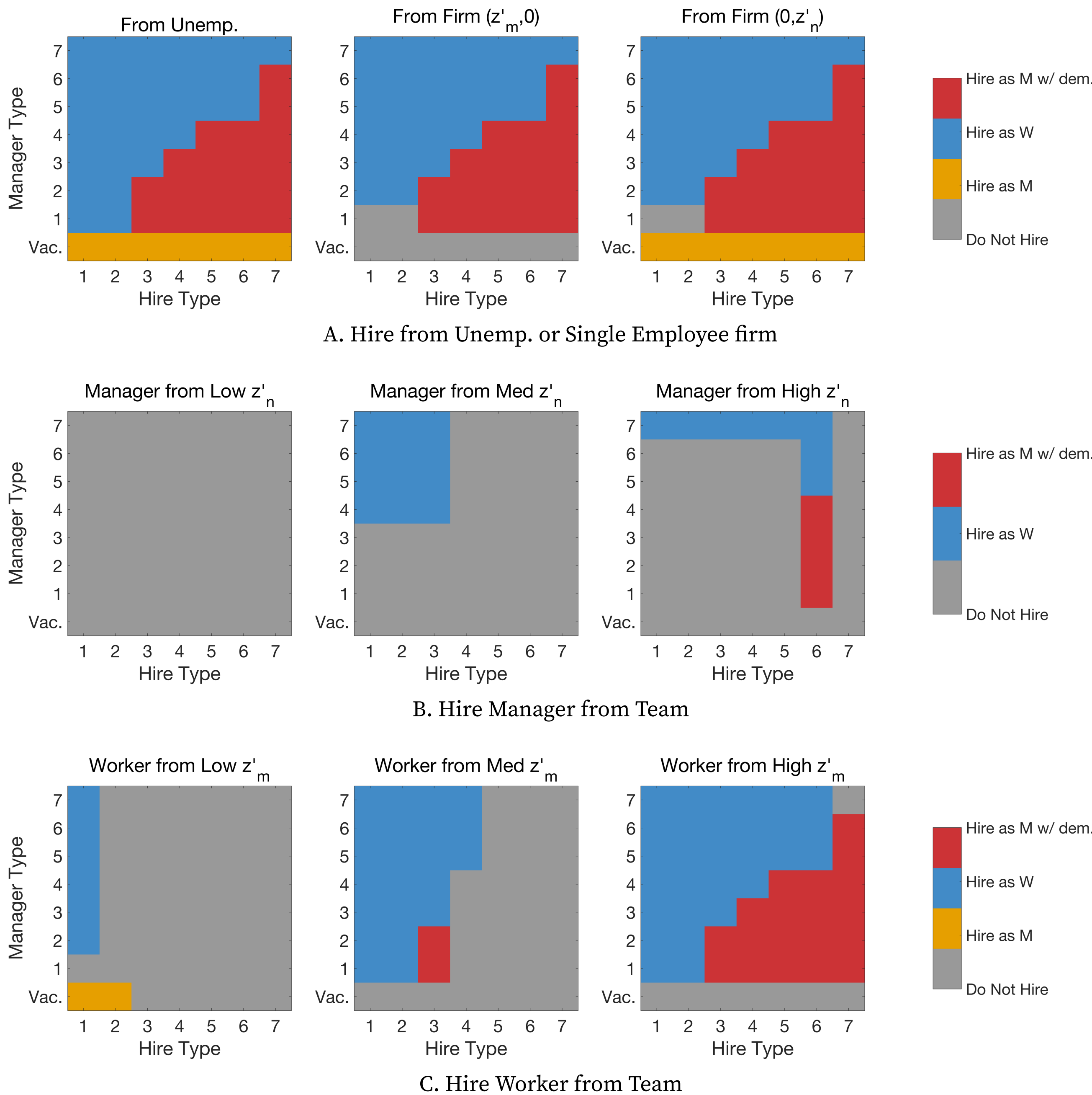
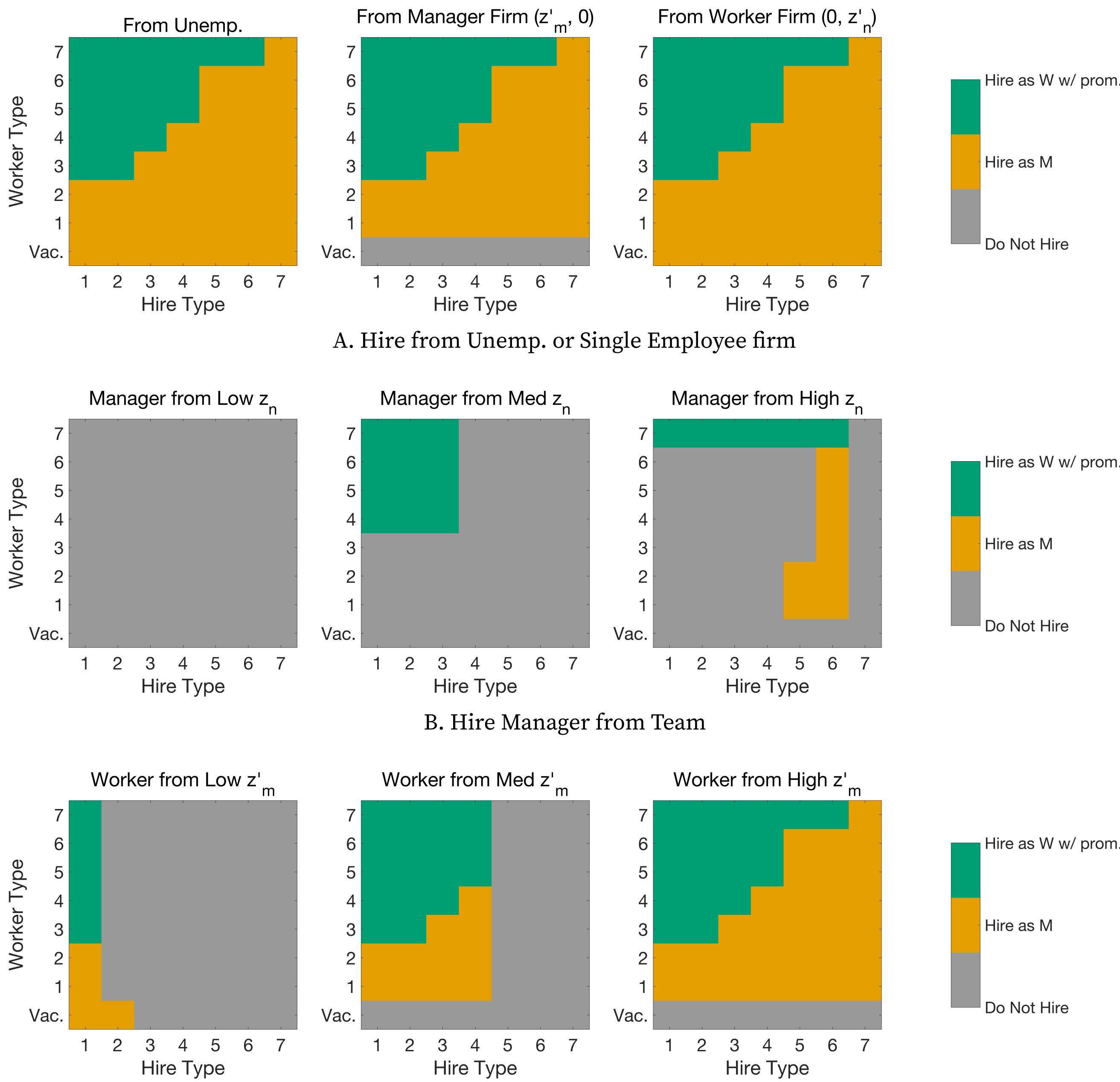


FIGURE 1. Hiring Policy Function of firm with manager only $(z_m, 0)$

incur. Second, we see an inaction region, where firms with very low managers cannot overcome the outside option of the counterpart firm and are not able to form a team.

Naturally, these inaction regions become larger on panels 1B and 1C, where we see the hiring policies of poaching managers and workers from teams, which have larger outside options. From row 1B we see that no firm poaches any type of manager from a team with low worker skill, as the continuation value of the poached team would be too



A. Hire from Unemp. or Single Employee firm

B. Hire Manager from Team

C. Hire Worker from Team

FIGURE 2. Hiring Policy Function of firm with worker only $(0, z_n)$

low. Breaking the team and keeping only a bad worker is costly in terms of continuation value, increasing the outside option of the poached firm and blocking the trades from occurring. When the worker type improves, we see some very good poaching firms being able to hire the manager from the team. The poached firm here suffer less than in the previous case as they would be able to replace the lost manager by promoting a better worker. Curiously, for a very high-skilled worker, some low-skilled firms manage to poach

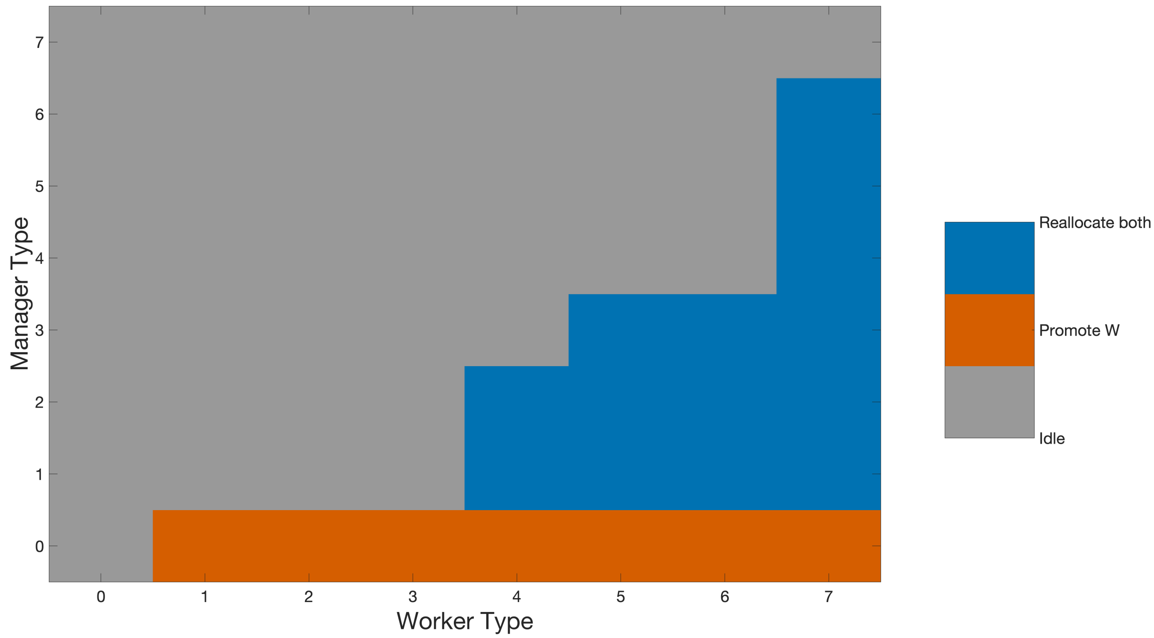


FIGURE 3. Reallocation Policy Functions

good managers, but in a non-monotonic fashion. If the manager is not too good, the poached firm is able to keep the team together, but if the manager is too skilled, the gains from poaching firms outweigh the outside options and hiring occurs. Finally, in 1C we see the hiring policies for getting a worker from a established team. Noticeably, the inaction regions are smaller here than in 1B, confirming the intuition that it is harder to poach a sitting manager due to the costs of promotions that the poached firm would have to incur to restore productivity. Starkly, in the last subplot we see that when the poached firm has a very high skilled manager, poaching of the workers always occurs against another firm. This the poaching firm to form a team while the poached firm retains high productivity.

Similarly, in Figure 2 we see the hiring and allocation policies of a firm with worker only $(0, z_n)$. The hiring patterns overall follow similar logic to the previous figure, with the inaction regions expanding as we move from hiring from the unemployment pool to poaching from teams, although non-monotonically. Important to this paper is the pattern and intensity of promotions that can happen because of the hiring policies. In all subplots we observe that promotions occur in the upper left corners, above the 45 degree line. When the hired type is relatively low compared to the sitting worker, the firm hires and promotes their incumbent employee to manager. Again, because of reallocation costs, the promotion regions are never in the entire region above the 45 degree line, even when hiring from unemployment. This leads to teams being formed with incumbent managers with lower skill than the new hire. In this case, firms wait for the worker to accumulate skill before promoting.

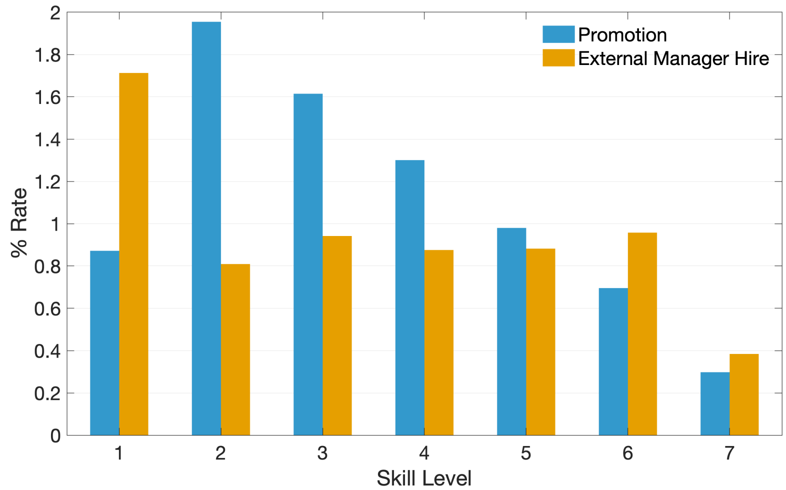


FIGURE 4. Internal and External Rates across Manager Skill Levels

In Figure 3 we have the reallocation policy for every type firm after the search and match and before the production stage. The manager types are on the vertical axis (including zeros for no manager) and workers' are in the horizontal axis, so that each cell is a possible type of firm in the economy. We see a very intuitive pattern where firms that end up with only a worker $(0, z_n)$ will always promote them to manager. This is not necessarily obvious, as the firm could have waited for the worker to accumulate more skill before promoting or even waited to find a manager in the external markets and saved on reallocation costs. Again, this suggests that in our quantification, the gains of making the firm operational today outpace the costs of promoting, even when considering future benefits of better matches. Moving up to teams, we observe that some firms below the 45 degree line will swap the manager with the worker, incurring both costs of promotion and demotion. Intuitively firms with very good workers, that just suffered from a learning shock, will choose to reallocate both agents prior to production.

All these policy patterns suggest that firms in the economy are moving away from being organized with only a worker at the production stage. Vacant firms allocate hires straight to manager and firms that suffered from managerial turnover are promoting the remaining worker. At the same time, teams are moving towards a more productive internal structure, either hiring relatively better agents as managers or promoting learning workers. This reallocation is not perfect due to the presence of reallocation costs.

4.2. Internal Market Mechanisms

In this subsection, we discuss mechanisms of internal labor markets present in the model. We begin by decomposing the internal and external rates of hiring into managerial positions across the skill distribution. Figure 4 shows, for each level of the skill ladder, the

TABLE 6. Varying Reallocation Costs, Relative to Baseline

% Baseline	No Costs	Double Costs	Double c_p	Double c_d	No Internal Markets
Manager					
Wage	-1.56	5.17	5.23	-1.83	3.74
Skill	-0.93	-9.68	-6.82	-3.03	-11.21
Worker					
Wage	-7.62	-9.70	-1.54	5.75	-25.19
Skill	0.14	-6.35	-6.84	-1.81	-8.91
Average Product	-0.80	-10.60	-7.29	-3.28	-12.80

proportion of managers at that skill level who entered through either internal promotion or external hiring. The figure reveals that external hiring rates are more prominent at the extremes of the skill distribution, while internal promotions are relatively more common for middle-skilled managers. The intuition is straightforward: firms look to external markets either to recruit low-skilled managers when vacant or to attract top-tier managers who cannot be easily found from within their ranks. In contrast, internal promotions for mid-level managers are more frequent, driven by skill accumulation among workers and the need to replace managers who have left the firm. This highlights that, while the average internal and external rates are similar, they operate in different parts of the skill distribution. This differentiation showcases the role of internal markets as a crucial mechanism for firms to harness skill accumulation and maintain high productivity in the event of managerial turnover.

To further explore how firms make use of internal labor markets, we present a sequence of exercises where the model is solved under varying levels of reallocation costs, with all other parameters held constant at their baseline calibration values. Table 6 shows the results of these alternative scenarios, detailing their effects on the average wages and skill levels of managers and workers. Additionally, we report the average product, which serves as a measure of overall productivity in the economy. All values are expressed as percentage deviations from the baseline calibration. Although the results depend on complex, general equilibrium outcomes, we provide some intuition for the main findings.

The first column of Table 6 represents the scenario with zero reallocation costs, allowing firms to freely reassign employees. In this case, the threshold regions discussed in Section 4.1 align with the 45-degree line. Notably, we observe a decline in both managers' wages and average skill levels, alongside a sharp decrease in workers' wages. This outcome reflects a shift in the composition of the managerial pool: without reallocation costs, firms promote a relatively larger share of middle-skilled workers to managerial roles, as shown in the first panel of Figure 5. Consequently, team formation becomes more frequent, but the average skill level of managers declines together with the average

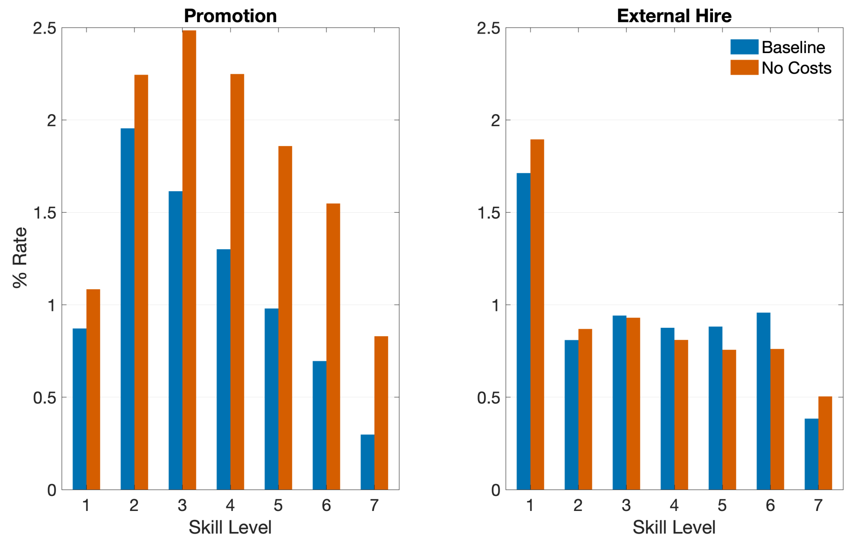


FIGURE 5. Internal and External Rates across Manager Skill Levels

quality of these teams. With a worse pool of managers, wages for those positions fall as hiring and poaching attempts increasingly target a less skilled set of agents.

The second to fourth columns of Table 6 show the effects of doubling both reallocation costs and the separate effects of doubling promotion and demotion costs. Qualitatively, increasing both costs produces similar results to an increase in promotion costs alone. Average skill levels for both managers and workers decline sharply, and the average product of the economy also falls. Intuitively, when promotion costs rise, it becomes harder for firms to organize teams with the optimal allocation of employees, making poaching by other firms more appealing since they can avoid promotion costs by directly allocating a new hire as manager. In equilibrium, firms with managers of lower skill than their workers bring down the average quality of both positions. Interestingly, manager wages increase with higher promotion costs, as the joint value of retaining a manager now includes the higher cost of promotion, making the sitting manager command a larger outside option. When demotion costs double, the wage effects are reversed, though quantitatively, all deviations from the baseline are less pronounced. This suggests that promotion costs naturally play a more significant role in the model, in terms of how firms more commonly are organized and the poaching patterns that emerge.

Finally, the last column of Table 6 shows the interesting case where reallocation costs are prohibitively high, shutting off the internal labor markets. Worker wages fall by a quarter, and their average skill level declines by nearly 9%, reflecting the reduced role of workers when firms cannot reallocate internally. Managerial skill also drops by 11%, due to the spillover effect of fewer workers accumulating skills and progressing up the internal ladder into managerial roles. Manager wages increase by 3.74%, following the same logic as in the high promotion cost scenario, though the effect is smaller due to the overall lower skill level of managers in the economy. Meanwhile, the average product falls sharply by 12.8%, indicating a significant productivity loss from the reduced roles of learning and internal reallocation.

This finding underscores a key point of this paper: internal markets serve as a substantial mechanism for talent allocation both within and across firms. In a setting with on-the-job learning, fluid internal reallocation provides firms incentives to form teams that foster skill accumulation, which then diffuses throughout the economy via external markets. Through a simple exercise, these figures provide an initial estimate of the impacts of internal markets on the economy, offering a benchmark to think of how interventions in improving internal mobility might influence aggregate productivity.

5. Non-Compete Clauses

In this section, we use the model to evaluate policies related to non-compete agreements (NCAs) within when firms have occupational structures and internal reallocation. Non-compete agreements, and their proposed bans, have drawn significant attention from U.S. policymakers, as approximately 18% of the country's workforce—including low-wage workers—are subject to clauses that limit job mobility.[3] In our setting, we will counterfactually introduce targeted NCAs to managers and workers separately, and study the differential effects of these policies on agents' wages, skill levels, and overall productivity, with a particular interest on the role of internal reallocation in firms' reactions to these changes in the environment.

Building on the baseline model in Section 2, we introduce NCAs in a straightforward manner. During a meeting , we assume that with probabilities φ_m and φ_n, the poached firm has a non-compete agreement in place for the manager and worker positions, respectively. In such cases, the poaching firm is prohibited from hiring the agent under the NCA, and the meeting ends without trade. Importantly, from the poached firm's perspective, a NCA prevents wage adjustments for the locked-in agent, as there is no credible external threat to induce a raise. This mechanism is key to understanding the impact of NCAs on wages. We separately adjust the probabilities φ_m and φ_n to target the average duration of 19 months for NCAs in the U.S. (Shi, 2023). The remaining parameters are kept at the baseline values for this counterfactual exercise.

Table 7 presents the results of the NCA counterfactuals, showing the percentage deviation from the baseline model for both the NCA Manager and NCA Worker cases. The primary observation is that wages decrease for both employee types under either counterfactual, with a sharper decline of 3.37% for workers when they are the target of the NVAs. These results are intuitive and consistent with the broader literature: the reduced

[3] See www.ftc.gov/noncompetes.

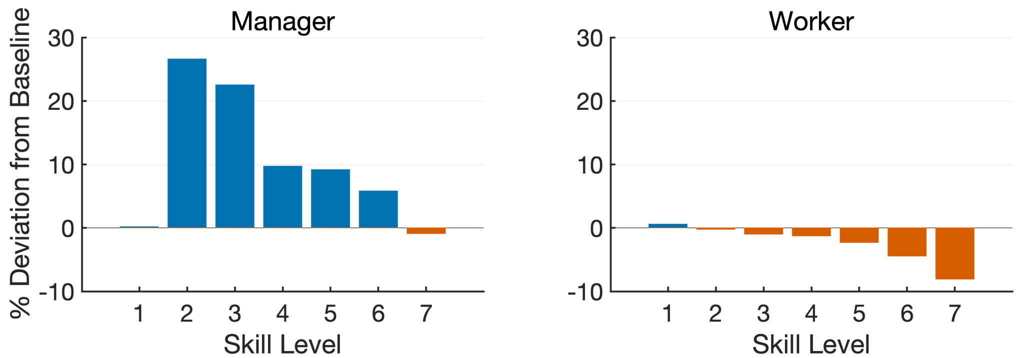
TABLE 7. Effects of Non-Compete Agreements Relative to Baseline

% Baseline	NCA Manager	NCA Worker
Manager		
Skill	1.37	0.21
Wage	-0.87	-1.05
Worker		
Skill	-4.77	1.21
Wage	-0.94	-3.37
Average Product	-1.42	0.70

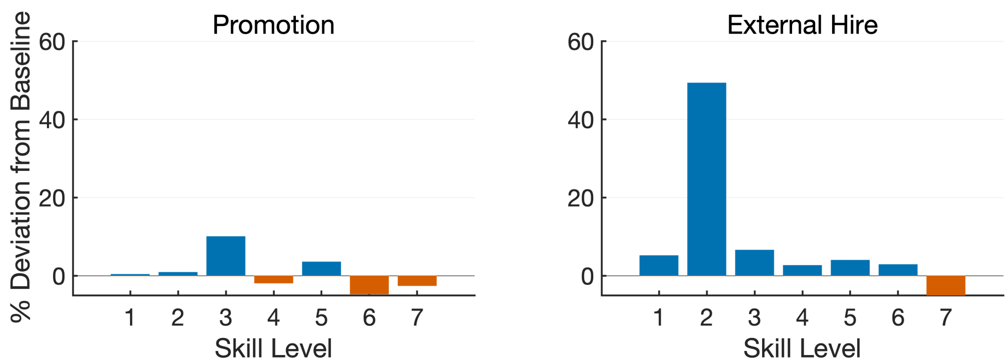
competitive pressure introduced by NCAs weakens the main driver of wage growth in a search model. With fewer outside threats, firms face less incentive to raise wages to retain employees, resulting in overall wage declines. The magnitude of these effects may also reflect general equilibrium dynamics, such as interactions with skill accumulation.

Examining the NCA for Managers in more detail, we observe that the skill level of managers increases by 1.37%, while the average skill level of workers drops by 4.77%. Figure 6A illustrates these changes in the distribution of skills for managers and workers across different rungs. The increase in managerial skill is driven by a shift in composition within the middle of the skill distribution, where employees more frequently transition into managerial roles under the counterfactual scenario. Due to the high density of agents at the upper levels of the skill ladder in our model, these compositional shifts end up increasing the average skill level in managerial positions. Conversely, worker skill declines broadly, with the largest decreases occurring at the top of their skill distribution. To support these facts, we look at panel 6B to notice how the internal and external rates of hiring into managerial positions change at each rung. We see that promotions become relatively more frequent for some middle-level skills, with external hiring following the pattern together with a spike in the second rung. In equilibrium, firms are promoting and hiring more aggressively to fill managerial positions with middle skilled agents. This shifts away skilled agents from worker positions and into managerial roles, which in turn hinders the skill accumulation of workers while on the job.

To understand these results, it is helpful to examine some of the underlying effects at play. First, in partial equilibrium, the joint value of a firm with only a manager $(z_m, 0)$ would *decrease* under a non-compete. The firm-employee unit is worse off, either because it becomes harder to poach managers locked into other firms, or because it cannot benefit from the joint value that poaching provides. In isolation, this would incentivize firms to



A. Manager and Worker Skill Distribution



B. Promotions and External Hires

FIGURE 6. NCA Manager Deviations from Baseline

move agents out of managerial roles, which contrasts with the final outcome we observe. This discrepancy suggests that general equilibrium forces, driven by shifts in firm distributions across the economy, are ultimately responsible for the higher promotion and external hiring rates for managers.

Intuitively, these forces can be broken down as follows. Firms with only a manager have a lower joint continuation value, yet they avoid demoting the manager without a new hire, thus incentivizing more aggressive team formation. Meanwhile, due to their lower value, firms with $(z_m, 0)$ are more vulnerable to poaching, prompting more external hires and some promotions triggered by those meetings. In turn, this increases worker turnover, which in equilibrium reduces the skill accumulation, as these agents are more frequently allocated to managerial positions within teams.

The first panel of Figure 7 shows the net differences in firm distribution across the economy relative to the baseline. This confirms our intuition: under the counterfactual with NCAs for managers, there are fewer firms with only managers across all levels of the skill ladder. Meanwhile, there is a relative increase in firms with teams, especially those paired with high-skilled managers. However, the increased turnover among workers hurts

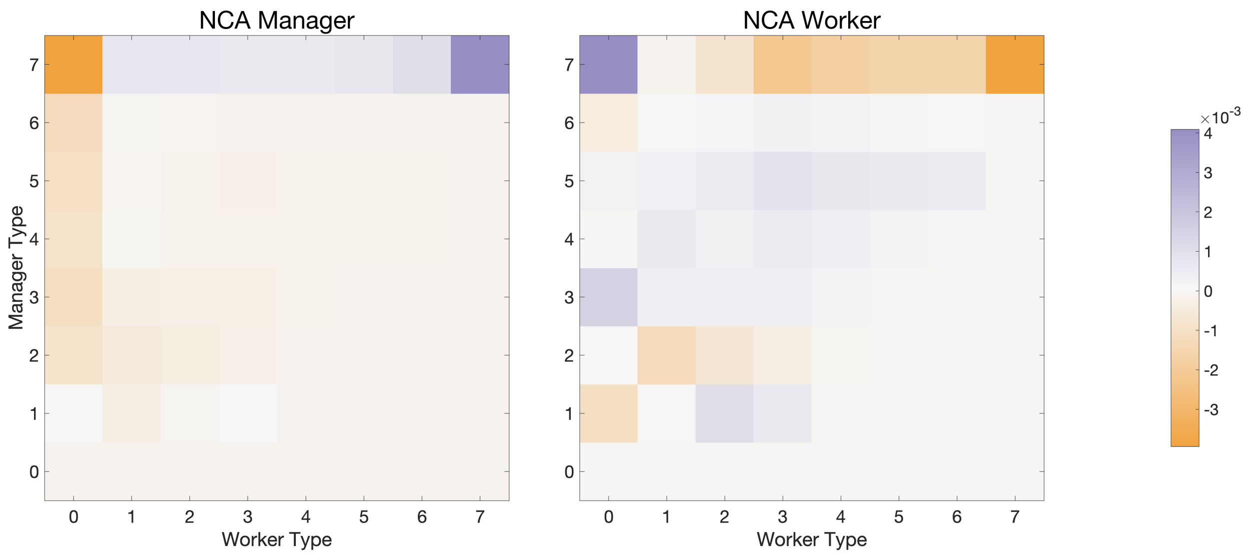
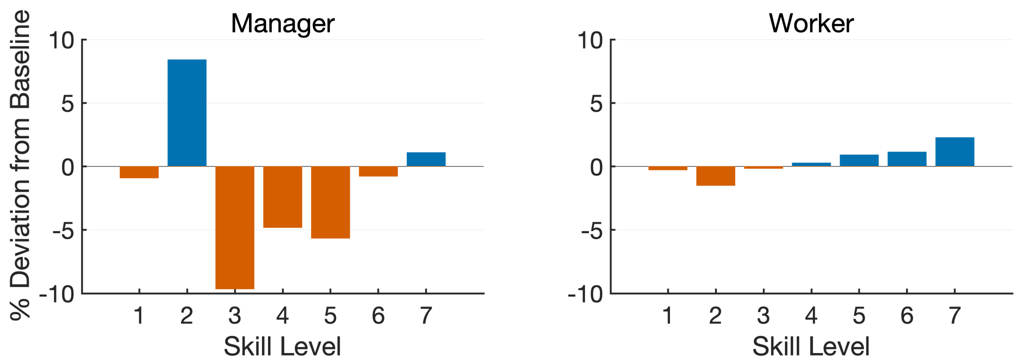


FIGURE 7. Net differences in Firm Distribution relative to Baseline

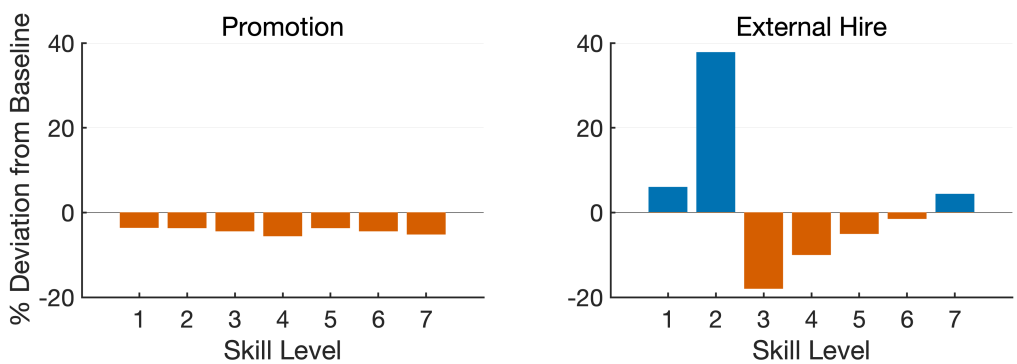
skill accumulation, and despite the rise in team formation, overall average productivity declines.

Turning to the NCA for Workers in Table 7, we observe a different set of outcomes.The average skill levels of both workers and managers improve, increasing by 1.21% and 0.21%, respectively. Figure 8 further decomposes these distributions and rates, showing an improvement in the skill distribution of workers, with an increase in higher-skilled types and a decrease in lower-skilled types. For managers, the composition shift is more nuanced: while there are fewer middle-skilled managers, the increase at the top skill tier is sufficient to slightly raise the average skill level. Looking at rates in Panel 8B, we see promotions falling somewhat uniformly across the board, while external hiring rates decrease for middle-skilled managers and increase for those in the top skill tier. In equilibrium, firms are promoting less and together with the fall of external hires for middle managers, we see an improvement in the skill accumulation of workers. This improvement then spills over to managers at the highest skill level.

Understanding these results requires again examining the general equilibrium forces at play. In partial equilibrium, the joint value of a firm with only a worker $(0, z_n)$ would again *decrease* under a non-compete. Firm-employee units would be incentivized to move workers out of the locked-in state, which would create incentives to increase promotions. This further incentivizes firms with a single employee to reorganize as firms with a manager, raising the mass of such firms, as seen in the second panel of Figure 7. Additionally, due to the lower continuation value associated with losing a manager, teams become more stable units under this counterfactual as their outside options improve. We observe a higher density of teams with mid-skilled combinations of employees. These stable units



A. Manager and Worker Skill Distribution



B. Promotions and External Hires

FIGURE 8. NCA Worker Deviations from Baseline

experience lower turnover, leading to fewer promotions. At the same time, the longer duration of workers within teams allows for greater skill accumulation, which, in equilibrium, benefits managers and raises the average productivity of the economy by 0.70%.

This exercise provides valuable quantification of non-compete effects on the economy, highlighting differential impacts on managers and workers and allowing us to examine the role of external and internal markets in shaping these outcomes. The key takeaway is that, while the conventional view—that NCAs erode wages by reducing competitiveness—remains relevant, the flow dynamics of external and internal reallocation are crucial in determining firm organization and the patterns of skill accumulation in the economy. When non-competes target managers—positions crucial for productivity—firms aggressively promote and hire workers into these top roles, which diverts skill away from positions that benefit from on-the-job learning. This shift can lead to a decline in the quality of agents in managerial roles and a reduction in aggregate productivity. Conversely, when firms have market power to retain workers— who learn and develop within the firm—turnover decreases, and fewer promotions are needed to replace separations. This results in more stable teams, increasing the aggregate supply of skilled agents and potentially boosting overall output.

6. Conclusion

This paper develops, characterizes and provides a quantification of a search model in which firms have occupational roles, can internally reallocate resources and workers cam accumulate skill while on the job. Focusing on managers, we investigate how firms find and allocate managerial talent through both internal promotions and external hires. In the data, both channels are substantial, highlighting the need to carefully consider the distinct incentives and interactions within each mechanism. Understanding the labor market dynamics for managers thus requires examining the interconnected roles of internal reallocation and external competition.

Our model is calibrated with detailed establishment-employee data from Germany and simultaneously captures the internal and external dynamics of this labor market, providing an ideal setting to study its intricate connections. The model is also able to replicate salient features in data regarding managerial the wage premium and the contribution to worker skill accumulation coming from managers in the same establishment.

The counterfactual analysis of Non-Compete Agreements highlights the significant effects that labor market policies have on firm organization and productivity. When NCAs restrict managerial mobility, firms respond in equilibrium by fast-tracking promotions, often filling managerial roles with less-experienced employees, which hinders skill accumulation and diminishes aggregate productivity. Conversely, NCAs targeting non-managerial positions reduce internal promotions, enabling skill accumulation among workers that ultimately spill over to managerial roles, raising overall productivity. These findings emphasize the need to consider internal reallocation when evaluating labor market policies, as restrictions in one reallocation channel can affect firms' incentives, its composition, and ultimately aggregate productivity.

Our work points to several avenues for future research that digs further into opening the firm's 'black-box' and understanding internal dynamics while taking into account the external labor markets. In the current setting, we abstract from firm size for tractability; however, internal markets might play a larger role in larger firms that benefit from finely tuning their internal organization. Additionally, the relationship between manager allocation and firm size raises interesting questions, as managers can directly impact a firm's span of control (Lucas, 1978). In a model with manager turnover, firms' choice of size might take into consideration future internal movement into managerial positions, providing new perspectives to be investigated regarding firm dynamics. A richer structure within firms would enable the study of wage composition and dispersion within the production unit, accounting for the varying contributions of different positions to output and how these roles may be substituted in external markets.

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Appendix A. Model Appendix

A.1. Policy Functions - Hiring

The gains from trade, pin down the hiring function at the SM stage. A firm y hires z' an unemployment worker z'

$$h_u(z'; y) = \mathbb{1}\{\nu_{z'}(y) - u_{z'} > 0\} \tag{A1}$$

where $\mathbb{1}$ is the indicator function and 1 denotes that the firm will hire the worker if the gains from trade are positive. Simmilary, y hires from firm with only manager $(z'_m, 0)$

$$h_m(z'_m; y) = \mathbb{1}\left\{\nu_{z'_m}(y) - u_m(z'_m, 0) > 0\right\} \tag{A2}$$

from firm with worker only $(0, z'_n)$

$$h_n(z'_n; y) = \mathbb{1}\left\{\nu_{z'_n}(y) - u_n(0, z') > 0\right\} \tag{A3}$$

From firm with Team (z'_m, z'_n), y hire the manager

$$h_m(z'_m, z'_n; y) = \mathbb{1}\left\{\nu_{z'_m}(y) - u_m(z'_m, z'_n) > \left[\nu_{z'_n}(y) - u_n(z'_m, z'_n)\right]^+\right\} \tag{A4}$$

alternatively , hire the worker

$$h_n(z'_m, z'_n; y) = \mathbb{1}\left\{\nu_{z'_n}(y) - u_n(z'_m, z'_n) > \left[\nu_{z'_m}(y) - u_m(z'_m, z'_n)\right]^+\right\} \tag{A5}$$

In equations (A4) and (A5), the firm will hire the agent that maximizes the gains from trade, which is the maximum between the two options of poaching the manager or the worker, conditional on the gains from trade being positive.

A.2. Policy Functions - Allocation Conditional on Hiring

Conditional on hiring the agent of skill level z', the firm decides where to allocate the employee based on continuation values $\widehat{V}$. Below we describe the allocation policy function, that are defined for each possible state of the firm, conditional on 'hiring' the agent z'.

A vacant firm $(0, 0)$ allocates the hire z' as manager if

$$p_m(z'; 0, 0) = \mathbb{1}\left\{\widehat{V}(z', 0) \geq \widehat{V}(0, z')\right\} \tag{A6}$$

and worker $p_n(z'; 0, 0) = 1 - p_m(z'; 0, 0)$

A firm with manager $(z_m, 0)$ allocates the hire z' as manager, firing the incumbent if

$$p_m^u(z'; z_m, 0) = \mathbb{1}\left\{\widehat{V}(z', 0) + U(z_m) > \max\left\{\widehat{V}(z_m, z'), \widehat{V}(z', z_m) - c_d\right\}\right\} \tag{A7}$$

Instead it chooses to allocate as manager, but demoting the incumbent

$$p_m^d(z'; z_m, 0) = \mathbb{1}\left\{\widehat{V}(z', z_m) - c_d > \max\left\{\widehat{V}(z_m, z'), \widehat{V}(z', 0) + U(z_m)\right\}\right\} \tag{A8}$$

and finally as a worker, in the emplty slot of the production unit, $p_n(z'; z_m, 0) = 1 - p_m^u(z'; z_m, 0) - p_m^d(z'; z_m, 0)$

A firm with worker only $(0, z_n)$ allocates the hire z' as worker, firing the incumbent if

$$p_n^u(z'; 0, z_n) = \mathbb{1}\left\{\widehat{V}(0, z') + U(z_n) > \max\left\{\widehat{V}(z', z_n), \widehat{V}(z_n, z') - c_p\right\}\right\} \tag{A9}$$

as worker, promoting the incument if

$$p_n^p(z'; 0, z_n) = \mathbb{1}\left\{\widehat{V}(z_n, z') - c_p > \max\left\{\widehat{V}(z', z_n), \widehat{V}(0, z') + U(z_n)\right\}\right\} \tag{A10}$$

and finally as a manager in the empty slot with the remaining $p_m^u(z'; 0, z_n) = 1 - p_n^p(z'; 0, z_n) - p_n^u(z'; 0, z_n)$

For firm with a team (z_m, z_n) we have to be more careful as any hire implies that one of the current employees have to be separated. The unit allocates the hire z' as a manager, firing the incumbent manager if

$$\begin{aligned} p_m^u(z'; z_m, z_n) = \mathbb{1}\Big\{&\widehat{V}(z', z_n) + U(z_m) > \\ &\max\left\{\widehat{V}(z_m, z') + U(z_n), \widehat{V}(z', z_m) + U(z_n) - c_d, \widehat{V}(z_n, z') + U(z_m) - c_p\right\}\Big\} \end{aligned} \tag{A11}$$

as a manager, demoting the current manager if

$$\begin{aligned} p_m^d(z'; z_m, z_n) = \mathbb{1}\Big\{&\widehat{V}(z', z_m) + U(z_n) - c_d > \\ &\max\left\{\widehat{V}(z', z_n) + U(z_m), \widehat{V}(z_m, z') + U(z_n), \widehat{V}(z_n, z') + U(z_m) - c_p\right\}\Big\} \end{aligned} \tag{A12}$$

The team may also allocate the fire as a worker, firing the incumbent one if

$$\begin{aligned} p_n^u(z'; z_m, z_n) = \mathbb{1}\Big\{&\widehat{V}(z_m, z') + U(z_n) > \\ &\max\left\{\widehat{V}(z', z_n) + U(z_m), \widehat{V}(z', z_m) + U(z_n) - c_d, \widehat{V}(z_n, z') + U(z_m) - c_p\right\}\Big\} \end{aligned} \tag{A13}$$

Instead as worker, promoting the current one

$$\text{(A14)} \quad p_n^p(z'; z_m, z_n) = \mathbb{1}\Big\{\widehat{V}(z_n, z') + U(z_m) - c_p > \max\Big\{\widehat{V}(z', z_n) + U(z_m), \widehat{V}(z', z_m) + U(z_n) - c_d, \widehat{V}(z_m, z') + U(z_n)\Big\}\Big\}$$

The above policy functions characterize the allocative firm decision in any state y for any level of skill of the hire z', condition of the hiring decision.

A.3. Policy Functions - Reallocation post-Search and Match

At the reallocation stage, the firm-employee units have to decide what to do with the current composition of the firm after having potentially suffered shocks or separations in the previous stages of the period. The policy functions are defined for each possible state of the firm, conditional on the continuation values unemployment U of production V that will take place immediately after the reallocation stage.

A firm with manager only $(z_m, 0)$

$$\text{(A15)} \quad d_m(z_m, 0) = \mathbb{1}\{V(0,0) + U(z_m) > \max\{V(0, z_m) - c_d, V(z_m, 0)\}\}$$

$$r_m(z_m, 0) = \mathbb{1}\{V(0, z_m) - c_d > \max\{V(0,0) + U(z_m), V(z_m, 0)\}\}$$

where the firmst line describes the decision to fire the manager, generating the joint value of $V(0,0) + U(z_m)$, while the second line describes the decision to reallocate the manager to a worker position, incuring a cost c_d. The firm also stays idle with probability $1 - d_m(z_m, 0) - r_m(z_m, 0)$.

Similarly, a firm with worker $(0, z_n)$ reallocation decisions are described by

$$\text{(A16)} \quad d_n(0, z_n) = \mathbb{1}\Big\{V(0,0) + U(z_n) > \max\Big\{V(0, z_n) - c_p, V(z_n, 0)\Big\}\Big\}$$

$$r_n(0, z_n) = \mathbb{1}\Big\{V(z_n, 0) - c_p > \max\{V(0,0) + U(z_n), V(0, z_n)\}\Big\}$$

which follows the same structure as above, the firm separate form the worker, promote the worker to manager at a cost c_p or stay idle with probability $1 - d_n(0, z_n) - r_n(0, z_n)$.

For the firm with team (z_m, z_n) it is convinient to write the policy functions with a slightly different logic. Instead of describing each possible mutually exclusive event, we write the policy functions for these events: the firm decides separates from the manager only; separates from the worker only; separate from both workers; reallocate the manager only or reallocate the worker only. Each of these policies is described below Firm fire the manager only

$$\text{(A17)} \quad d_m(z_m, z_n) = \mathbb{1}\Big\{\max\Big\{V(0, z_n) + U(z_m), V(z_n, 0) + U(z_m) - c_p\Big\}$$

$$> \max \left\{ V(z_m, z_n), V(z_m, 0) + U(z_n), V(0, z_m) + U(z_n) - c_d, \right.$$
$$\left. V(z_n, z_m) - c_d - c_p, V(0,0) + U(z_m) + U(z_n) \right\} \}$$

Notice that on the left-hand side within the max we have all instances where the firm would separate only from the manger, where either it si a simple separation with value $V(0, z_n) + U(z_m)$ or it also involves a reallocation of the worker, which is the value $V(z_n, 0) + U(z_m) - c_p$. On the right-hand side of the max we have all possible events where the firm is not separating from the manager only. Following the same logic the firm fire worker only if

$$d_n(z_m, z_n) = \mathbb{1}\{\max\{V(z_m, 0) + U(z_n), V(0, z_m) + U(z_n) - c_d\} \\ > \max \left\{ V(z_m, z_n), V(z_n, 0) + U(z_m), V(0, z_n) + U(z_m) - c_p, \right. \\ \left. V(z_n, z_m) - c_d - c_p, V(0,0) + U(z_m) + U(z_n) \right\} \} \tag{A18}$$

When the firm fires both agents we have

$$d_{mn}(z_m, z_n) = \mathbb{1}\{\max\{V(0,0) + U(z_m) + U(z_n)\} > \max\{\dots\}\} \tag{A19}$$

where again, the right-hand side inside the max contains all possible events where the firm does not separate from both agents. The firm will reallocate only the manager if

$$r_m(z_m, z_n) = \mathbb{1}\left\{\max\left\{V(0, z_m) + U(z_n) - c_d, V(z_n, z_m) - c_p - c_d\right\} > \max\{\dots\}\right\} \tag{A20}$$

and only the worker if

$$r_n(z_m, z_n) = \mathbb{1}\left\{\max\left\{V(z_n, 0) + U(z_m) - c_p, V(z_n, z_m) - c_p - c_d\right\} > \max\{\dots\}\right\} \tag{A21}$$

With the policies in equations (A17)-(A21) we can fully describe any reallocation decision for the firm with a team. For instance the firm decide to fire the manager and promote the worker if and only if $d_m(z_m, z_n) \cdot r_n(z_m, z_n) = 1$. As before the firm stays idle with the remaining g probability of all above described events

A.4. Stationary Distribution and Law of Motions

We will now describe the law of motions for the stationary distribution of the model implied by the policy functions above. As before, it is convenient to write the distributions at the beginning of each stage; search and match, reallocation and production.

A.4.1. Distributions after the Search and Match Stage

We describe the Law of motions for the distributions of firms and unemployed workers at each stage of the period. Let $e_u(z)$ be the distribution of unemployed workers, $e(y)$ the distribution of firms at the beginning of the Search and Match stage, this is right after the firm just produced and suffered from the set of shocks. Let $e_{1u}(z)$ be the distribution of unemployed workers implied by the Search and Match events and same for $e_1(y)$ for the firms.

Some notation to simplify the cumbersome expressions. Let $\mathcal{Y}_m(z)$ be the set of firms with a manager in state z and non-empty worker state, with a typical element being $y_m = (z, q)$, where q is the worker state. Similarly, let $\mathcal{Y}_n(z)$ be the set of firms with a worker in state z and non-empty manager state, with a typical element being $y_n = (q, z)$. The mass of unemployed of type z at the end of SM is given by

$$e_{1u}(z) = e_u(z)\left\{1 - \sum_y \frac{\lambda_u e(y)}{N} h_u(z; y)\right\}$$

(△)

$$+ e(z,0)\left\{\delta + \sum_{z'} \frac{\lambda_u e_u(z')}{N} h_u(z'; z)\, p_m^u(z'; z) + \sum_{z'_m} \frac{\lambda e(z'_m, 0)}{N} h_m(z'_m; z)\, p_m^u(z'_m; z)\right.$$

$$\left. + \sum_{z'_n} \frac{\lambda e(0, z'_n)}{N} h_n(z'_n; z)\, p_m^u(z'_n; z) + \sum_{y'_t} \frac{\lambda e(y'_t)}{N} \left[h_m(y'_t; z)\, p_m^u(z'_m; z) + h_n(y'_t; z)\, p_m^u(z'_n; z)\right]\right\}$$

(□)

$$+ e(0,z)\left\{\delta + \sum_{z'} \frac{\lambda_u e_u(z')}{N} h_u(z'; z)\, p_n^u(z'; z) + \sum_{z'_m} \frac{\lambda e(z'_m, 0)}{N} h_m(z'_m; z)\, p_n^u(z'_m; z)\right.$$

$$\left. + \sum_{z'_n} \frac{\lambda e(0, z'_n)}{N} h_n(z'_n; z)\, p_n^u(z'_n; z) + \sum_{y'_t} \frac{\lambda e(y'_t)}{N} \left[h_m(y'_t; z)\, p_n^u(z'_m; z) + h_n(y'_t; z)\, p_n^u(z'_n; z)\right]\right\}$$

$$+ \sum_{y_m \in \mathcal{Y}_m(z)} e(y_m)\left\{\delta + \sum_{z'} \frac{\lambda_u e_u(z')}{N} h_u(z'; y_m)\left(p_m^u(z'; y_m) + p_n^p(z'; y_m)\right)\right.$$

(○)

$$+ \sum_{z'_m} \frac{\lambda e(z'_m, 0)}{N} h_m(z'_m; y_m)\left(p_m^u(z'_m; y_m) + p_n^p(z'_m; y_m)\right)$$

$$+ \sum_{z'_n} \frac{\lambda e(0, z'_n)}{N} h_n(z'_n; y_m)\left(p_m^u(z'_n; y_m) + p_n^p(z'_n; y_m)\right)$$

$$+ \sum_{y'_t} \frac{\lambda e(y'_t)}{N} \left[h_m(y'_t; y_m) \left(p^u_m(z'_m; y_m) + p^p_n(z'_m; y_m) \right) + h_n(y'_t; y_m) \left(p^u_m(z'_n; y_m) + p^p_n(z'_n; y_m) \right) \right] \Bigg\}$$

(◇)

$$+ \sum_{y_n \in \mathcal{Y}_n(z)} e(y_n) \Bigg\{ \delta + \sum_{z'} \frac{\lambda_u e_u(z')}{N} h_u(z'; y_n) \left(p^u_n(z'; y_n) + p^p_m(z'; y_n) \right)$$

$$+ \sum_{z'_m} \frac{\lambda e(z'_m, 0)}{N} h_m(z'_m; y_n) \left(p^u_n(z'_m; y_n) + p^p_m(z'_m; y_n) \right)$$

$$+ \sum_{z'_n} \frac{\lambda e(0, z'_n)}{N} h_n(z'_n; y_n) \left(p^u_n(z'_n; y_n) + p^p_m(z'_n; y_n) \right)$$

$$+ \sum_{y'_t} \frac{\lambda e(y'_t)}{N} \left[h_m(y'_t; y_n) \left(p^u_n(z'_m; y_n) + p^p_m(z'_m; y_n) \right) + h_n(y'_t; y_n) \left(p^u_n(z'_n; y_n) + p^p_m(z'_n; y_n) \right) \right] \Bigg\}$$

The first line describes the mass that stays in unemployment, net of any hiring from the unemployed pool of z. The block that starts at (△) describes the mass entering unemployment coming from a firm $(z, 0)$, either exogenous (δ) or if the firm $(z, 0)$ replace-hire its manager by someone else (p^u_m). Similarly, the block that at (□) describes the mass entering unemployment coming from a firm $(0, z)$, either exogenous (δ) or if the firm $(0, z)$ replace-hire its worker by someone else (p^u_n). The blocks that start at (○) and (◇) describe the mass entering unemployment coming from firms with full teams. The only difference in this case is that we have to consider the firings of z that comes accompanied by either demotion (p^d_m) or promotion (p^p_n).

Now for the mass of firms at the end of the SM stage. Let

$$\begin{aligned} H(y) = & \sum_{z'} \frac{\lambda_u e_u(z')}{N} h_u(z'; y) \\ & + \sum_{z'_m} \frac{\lambda e(z'_m, 0)}{N} h_m(z'_m; y) \\ & + \sum_{z'_n} \frac{\lambda e(0, z'_n)}{N} h_n(z'_n; y) \\ & + \sum_{y'_t} \frac{\lambda e(y'_t)}{N} \left[h_m(y'_t; y) + h_n(y'_t; y) \right] \end{aligned}$$

This is the total probability of any firm at y hire any agent. This will be useful when writing the mass of firms that leave each state. Additionally, let:

$$\mathcal{P}_m(z_m, z_n) = \sum_{y'} \frac{\lambda e(y')}{N} \left[h_m(z_m, z_n; y') \right]$$

be the probability that a firm with has its manager poached by any other firm y' in the economy. Similarly, let

$$\mathcal{P}_n(z_m, z_n) = \sum_{y'} \frac{\lambda e(y')}{N} \left[h_n(z_m, z_n; y') \right]$$

be the probability that a firm with has its worker poached by any other firm y' in the economy.

The mass of vacant firms $(0, 0)$ at the end of the SM stage is given by:

$$\begin{aligned} e_1(0,0) = {} & e(0,0)\left[1 - H(0,0)\right] \\ & + \sum_{z'} e(z',0) \left\{ \delta + \sum_{y} \tfrac{\lambda e(y)}{N} h_m(z'; y) \right\} \\ & + \sum_{z'_n} e(0,z'_n) \left\{ \delta + \sum_{y} \tfrac{\lambda e(y)}{N} h_n(z'_n; y) \right\} \end{aligned}$$

The first line describes the mass that stays vacant, net of any hiring from any source. The second line describes the mass of all firms with a manager in state z' lost their manager either exogenously or by poaching. The third line describes the mass of all firms with a worker in state z'_n lost their worker either exogenously or by poaching.

For firms with a manager $(z, 0)$, the mass at the end of the SM stage is given by:

$$\begin{aligned} e_1(z,0) = {} & e(z,0)\left[1 - \delta - H(z) - \mathcal{P}_m(z,0)\right] \\ & + e_u(z) \left[\frac{\lambda_u e(0,0)}{N} h_u(z;0,0)\, p_m(z;0,0) + \sum_{z'_m} \frac{\lambda_u e(z'_m,0)}{N} h_u(z;z'_m,0)\, p^u_m(z;z'_m) \right] \\ & + \sum_q e(z,q) \left[\delta + \mathcal{P}_n(z,q) + \tfrac{\lambda e(0,0)}{N} h_m(z,q;0,0)\, p_m(z;0,0) + \sum_{z'_m} \tfrac{\lambda e(z'_m,0)}{N} h_m(z,q;z'_m,0)\, p^u_m(z;z'_m,0) \right] \\ & + \sum_q e(q,z) \left[\tfrac{\lambda e(0,0)}{N} h_n(q,z;0,0)\, p_m(z;0,0) + \sum_{z'_m} \tfrac{\lambda e(z'_m,0)}{N} h_n(q,z;z'_m,0)\, p^u_m(z;z'_m,0) \right] \end{aligned}$$

The first line describes the mass that stays in the state $(z, 0)$, net of any hiring from any source and from the probability of being poached by any source. The second line describes inflows from unemployment, when an agent of type z is hired either by a vacant firm ans set as manager of by another firm with manager, that replace-hires its own manager. The third line describes inflows from full teams firms that either loses their worker q - exogenously or by poaching, or that loses their manager z to an empty firm of to a firm that replaces-hires its manager; in either case creating another $(z, 0)$ firm. The last line describes inflows from full teams firms that have the agent z as a worker. IN this case the inflows into $(z, 0)$ occur when empty firms or firms with managers replace-hire their own manager.

For firms with a worker $(0, q)$, for any q the mass at the end of the SM stage is given by:

$$\begin{aligned} e_1(0,q) &= e(0,q)\left[1-\delta-H(0,q)-\mathcal{P}_n(0,q)\right] \\ &+ e_u(q)\left[\frac{\lambda_u e(0,0)}{N} h_u(q;0,0)\, p_n(q;0,0) + \sum_{z'_n} \frac{\lambda_u e(0,z'_n)}{N} h_u(q;0,z'_n)\, p^u_n(q;0,z'_n)\right] \\ &+ \sum_z e(z,q)\left[\delta + \mathcal{P}_m(z,q) + \frac{\lambda e(0,0)}{N} h_n(z,q;0,0)\, p_n(q;0,0) + \sum_{z'_n} \frac{\lambda e(0,z'_n)}{N} h_n(z,q;0,z'_n)\, p^u_n(q;0,z'_n)\right] \\ &+ \sum_z e(q,z)\left[\frac{\lambda e(0,0)}{N} h_m(q,z;0,0)\, p_n(q;0,0) + \sum_{z'_n} \frac{\lambda e(0,z'_n)}{N} h_m(q,z;0,z'_n)\, p^u_n(q;0,z'_n)\right] \end{aligned}$$

Where the logic mirrors the one from the previous case. Finally for firm with teams (z, q), one last piece of notation to make this complex case a bit more manageable. Let $\Gamma_{z,y',j}(y)$ be the total probability of a firm reaching state y after hiring an agent of type z from firm y' to be at occupation $j = \{m, n\}$ denoting manager and worker respectively. This is given by:

$$\begin{aligned} e_1(z,q) &= e(z,q)\left\{1-2\delta-H(z,q)-\mathcal{P}_m(z,q)-\mathcal{P}_n(z,q)\right\} \\ &+ e_u(z)\Gamma_{z,u,m}(z,q) + e_u(q)\Gamma_{q,u,n}(z,q) \\ &+ \sum_{y_m \in \mathcal{Y}_m(z)} e(y_m)\Gamma_{z,y_m,m}(z,q) + \sum_{y_n \in \mathcal{Y}_n(z)} e(y_n)\Gamma_{z,y_n,m}(z,q) \\ &+ \sum_{y_m \in \mathcal{Y}_m(q)} e(y_m)\Gamma_{q,y_m,n}(z,q) + \sum_{y_n \in \mathcal{Y}_n(q)} e(y_n)\Gamma_{q,y_n,n}(z,q) \end{aligned}$$

The first line describes the mass that stays in (z, q), net of exogenous separations, any hiring and any poaching, as any of these events would mean a firm left the state. The second line describes inflows from unemployment, when an agent of type z or q are hired and correctly allocated to form a team. (z, q). The third line describes all the possible ways a firm can find an agent z currently allocated in a firm as a manager $y_m \in \mathcal{Y}_m(z)$ or as a worker $y_n \in \mathcal{Y}_n(z)$, and the last line describes all the possible ways a firm can find an agent q currently allocated in a firm as a manager $y_m \in \mathcal{Y}_m(q)$ or as a worker $y_n \in \mathcal{Y}_n(q)$. In both cases the poaching firm hires the agent and allocated it in such way to form a team (z, q).

A.4.2. Distributions after Reallocation Stage

Take as given the distributions $e_{1u}(z)$ and $e_1(y)$, we can derive the law of motions implied by the reallocation policies that take place before production. Let $e_{2u}(z)$ be the distribution of unemployed workers implied by the the reallocation events and same for $e_2(y)$ for the firms.

The mass of unemployed of type z at the end of the reallocation stage is given by

$$\begin{aligned} e_{2u}(z) = e_{1u}(z) + e_1(z,0)d_m(z,0) + e_1(0,z)d_n(0,z) \\ + \sum_q \left[e_1(z,q)\left(d_m(z,q) + d_{mn}(z,q)\right) \right. \\ \left. + \; e_1(q,z)\left(d_n(q,z) + d_{mn}(q,z)\right) \right] \end{aligned}$$

The first line describes the existing mass of unemployed agents at z added by single agents firms that decide to separate from their manager or worker. The second and thirs lines follows the same logic for firms with a full team that have z as either their manager or worker and decide to separate from them, leading into unemployment.

The mass of vacant firms $(0, 0)$ at the end of the reallocation stage is given by:

$$e_2(0,0) = e_1(0,0) + \sum_z \left[e_1(z,0)d_m(z,0) + e_1(0,z)d_n(0,z) + \sum_q e_1(z,q)d_m n(z,q) \right]$$

More straightforwardly, it sis the existing mass of vacan firms added by firms with single agents that decide to separate or from full teams that decide to separate from both agents.

For firms with a manager $(z, 0)$, the mass at the end of the reallocation stage is given by:

$$\begin{aligned} e_2(z,0) = e_1(z,0)\left[1 - d_m(z,0) - r_m(z,0)\right] + e_1(0,z)r_n(0,z) \\ + \sum_q \left[e_1(z,q)d_n(z,q)\left(1 - r_m(z,q)\right) + e_1(q,z)d_m(q,z)r_n(q,z) \right] \end{aligned}$$

The first lines display the existing mass of firms at $(z, 0)$ that stays in the state, net of any separation or reallocation, that would change the state. Added to that we have the mass of firms where z is the worker that decide to promote the worker to manager. The last line describes the mass of firms with a full team that decide to separate from one of its workers q and keep z as a manager, entailing a firm with only manager $(z, 0)$.

For firms with a worker $(0, q)$, for any q the mass at the end of the reallocation stage is given by:

$$\begin{aligned} e_2(0,q) &= e_1(0,q)\left[1 - d_n(0,q) - r_n(0,q)\right] + e_1(z,0)r_m(z,0) \\ &+ \sum_z \left[e_1(z,q)d_m(z,q)\left(1 - r_n(z,q)\right) + e_1(q,z)d_n(q,z)r_m(q,z)\right] \end{aligned}$$

following the same structure from the firms with only a manager case. Finally, for the full team firm (z, q), the mass at the end of the reallocation stage is given by:

$$e_2(z,q) = e_1(z,q)\left[1 - d_m(z,q) - d_n(z,q) - r_m(z,q) - r_n(z,q) - d_{mn}(z,q)\right] + e_1(q,z)r_m(q,z)r_n(q,z)$$

Which is the mass of the same full teams (z, q) that stays idle plus the mass of full teams (q, z) that reallocate both agents without separations.

A.4.3. Distribution after Production, Shocks and Entry and Exit of Agents

After production firms are hit with shocks, namely the workers learn while paired with managers and unemployed workers have their skill depreciated stochastically. On top of that agents leave the economy with probability σ and are reborn into unemployment following a distribution $\pi(z) \geq 0$ with $\sum_z \pi(z) = 1$.

It is convenient to split this into two final steps. Let $e_{3u}(z)$ be the distribution of unemployed workers after the shocks but before the entry and exit of agents, $e_3(y)$ for the firms. The mass of unemployed of type z at the end of the period is given by

$$\begin{aligned} e_{3u}(z) &= sum_{z'} Q_u(z|z')e_{2u} \\ e_3(z,0) &= e_2(z,0) \\ e_3(0,q) &= e_2(0,q) \\ e_3(z,q) &= \sum_{q'} Q(q|q',z)e_2(z,q') \end{aligned}$$

where, on the last equation the mass of firms at (z, q) depens on how many firms at (z, q') suffered the shock to transition to (z, q).

Finally, let $e_{+u}(z)$ be the distribution of unemployed workers after entry and exit of agents, $e_+(y)$ for the firms. The mass of unemployed of type z that will be relevant for next period Search and Match is given by:

$$e_{+u}(z) = (1-\sigma)e_{3u}(z) + \sigma\pi(z)$$

$$e_{+}(0,0) = e_3(0,0)$$

$$e_{+}(z,0) = (1-\sigma)e_3(z,0) + \sigma(1-\sigma)\sum_q e_3(z,q)$$

$$e_{+}(0,q) = (1-\sigma)e_3(0,q) + \sigma(1-\sigma)\sum_z e_3(z,q)$$

$$e_{+}(z,q) = (1-\sigma)^2 e_3(z,q)$$

The distribution is stationary if and only if $e_{+u}(z) = e_u(z)$ for every skill level z and $e_{+}(y) = e(y)$ for every state y. The stationary distributions are a key part of the equilibrium, as we need to guarantee that the policy functions are consistent with the stationary distribution.

Appendix B. Worker Value Functions

Given the equilibrium value and policy functions, we can write the agent's value functions The share of the worker in the surplus is delivered by a wage that remains constant, *unless* to induce a correct allocation across employment states As in Herkenhoff et al. (2024) wage only changes when: The agent is poached by another firm, gets its Mg Value at the new firm The incumbent firm keeps the agent after a poaching attempt, raises the wage to match the agent's outside option The current value of the agent falls below the Mg Value of the agent to the firm at its current position

The first two conditions raise wages to ensure the correct allocation of the agent across firms The third condition lowers the wage, adjusting for the worker's actual marginal value to the firm. This ensures that agents always have incentives to report offers truthfully to the team. Assumptions are in line with the search framework and to some extent with the data Lack of competitive pressure on wages makes them sticky, and firms find optimal to adjust only when necessary

Important caveat: promotions are not immediately priced in the wage, but are in the continuation value of the agent When approached by another firm, the incumbent will value the promoted agetn as its Mg Value is higher How does that square off with data? Not sure and we are ill-suited to answer that question with the data we have On a broad sense it might be consistent with the apparent fact that internal promotions have a smaller premium than external hires into manager The alternative would be some ad-hoc assumption on how the wage change upon promotion, that would lack some contractual foundation (I think so) Since this paper is not about promotion premium we might be better off being a bit more agnostic about it, and sticking to what makes sense in a search framework

For ease of notation I am dropping the state of the hiring firm when it is easy to be inferred from the context. For a manager in a firm $(z_m, 0)$ Let

$$\begin{aligned}\mathbb{W}_m(z', w, z_m, 0) &= p_m^u(z')U(z_m) + p_m^d(z')\widehat{W}_n(w, z', z_m) \\ &\quad + \left(1 - p_m^u(z') - p_m^d(z')\right)\widehat{W}_m(w, z_m, z') - \widehat{W}_m(w, z_m, 0)\end{aligned}$$

This term captures the continuation value for the manager currently at $(z_m, 0)$ being paid w upon hiring any z', considering all possible allocations in the current firm. The manager value function at $(z_m, 0)$ is then:

$$\begin{aligned}\widetilde{W}_m(w, z_m, 0) &= \widehat{W}_m(w, z_m, 0) + \delta\left[U(z_m) - \widehat{W}_m(w, z_m, 0)\right] \\ &\quad + \sum_{z'} \frac{\lambda_u e_u(z')}{N} h_u(z'; z_m, 0)\mathbb{W}_m(z', w, z_m, 0) \\ &\quad + \sum_{z'_m} \frac{\lambda e(z'_m, 0)}{N} h_m(z'_m; z_m, 0)\mathbb{W}_m(z'_m, w, z_m, 0) \\ &\quad + \sum_{z'_n} \frac{\lambda e(0, z'_n)}{N} h_n(z'_n; z_m, 0)\mathbb{W}_m(z'_n, w, z_m, 0) \\ &\quad + \sum_{y'_t} \frac{\lambda e(y'_t)}{N}\left[h_m(z'_m; z_m, 0)\mathbb{W}_m(z'_m, w, z_m, 0) + h_n(z'_n; z_m, 0)\mathbb{W}_m(z'_n, w, z_m, 0)\right] \\ &\quad + \sum_{y} \frac{\lambda e(y)}{N}\left[\max\left\{\widehat{W}_m(w, z_m, 0), \min\left\{\nu_{z_m}(y), \gamma\nu_{z_m}(y) + (1-\gamma)u_m(z_m, 0)\right\}\right\}\right. \\ &\qquad \left. - \widehat{W}_m(w, z_m, 0)\right]\end{aligned}$$

Analogously for a worker in a firm $(0, z_n)$ we can define

$$\begin{aligned}\mathbb{W}_n(z', w, 0, z_n) &= p_n^u(z')U(z_n) + p_n^p(z')\widehat{W}_m(w, z_n, z') \\ &\quad + \left(1 - p_n^u(z') - p_n^p(z')\right)\widehat{W}_n(w, z', z_n) - \widehat{W}_n(w, 0, z_n)\end{aligned}$$

This is the continuation value for the worker currently at $(0, z_n)$ being paid w upon hiring any z', considering all possible allocations in the current firm. The worker value function at $(0, z_n)$ is then:

$$\widetilde{W}_n(w, 0, z_n) = \widehat{W}_n(w, 0, z_n) + \delta\left[U(z_n) - \widehat{W}_n(w, 0, z_n)\right]$$

$$+\sum_{z'}\frac{\lambda_u e_u(z')}{N}h_u(z';0,z_n)\mathbb{W}_n(z',w,0,z_n)$$

$$+\sum_{z'_m}\frac{\lambda e(z'_m,0)}{N}h_m(z'_m;0,z_n)\mathbb{W}_n(z'_m,w,0,z_n)$$

$$+\sum_{z'_n}\frac{\lambda e(0,z'_n)}{N}h_n(z'_n;0,z_n)\mathbb{W}_n(z'_n,w,0,z_n)$$

$$+\sum_{y'_t}\frac{\lambda e(y'_t)}{N}\left[h_m(y'_t;0,z_n)\mathbb{W}_n(z'_m,w,0,z_n)+h_n(y'_t;0,z_n)\mathbb{W}_n(z'_n,w,0,z_n)\right]$$

$$+\sum_{y}\frac{\lambda e(y)}{N}\left[\max\left\{\widehat{W}_n(w,0,z_n),\min\left\{\nu_{z_n}(y),\gamma\nu_{z_n}(y)+(1-\gamma)u_n(0,z_n)\right\}\right\}\right.$$
$$\left.-\widehat{W}_n(w,0,z_n)\right]$$

As for agents with coworkers we can write the value of a manager in a firm (z_m,z_n) that is receiving wage w as:

$$\widetilde{W}_m(w,z_m,z_n)=\widehat{W}_m(w,z_m,z_n)+\delta\left[U(z_m)-\widehat{W}_m(w,z_m,z_n)\right]$$
$$+\left(\delta+\sum_{y}\frac{\lambda e(y)}{N}h_n(z_m,z_n;y)\left[\widehat{W}_m(w,z_m,0)-\widehat{W}_m(w,z_m,z_n)\right]\right)$$
$$+\sum_{z'}\frac{\lambda_u e_u(z')}{N}h_u(z';z_m,z_n)\mathbb{W}_m(z',w,z_m,z_n)$$
$$+\sum_{z'_m}\frac{\lambda e(z'_m,0)}{N}h_m(z'_m;z_m,z_n)\mathbb{W}_m(z'_m,w,z_m,z_n)$$
$$+\sum_{z'_n}\frac{\lambda e(0,z'_n)}{N}h_n(z'_n;z_m,z_n)\mathbb{W}_m(q',w,z_m,z_n)$$
$$+\sum_{y'_t}\frac{\lambda e(y'_t)}{N}\left[h_m(y'_t;z_m,z_n)\mathbb{W}_m(z'_m,w,z_m,z_n)+h_n(y'_t;z_m,z_n)\mathbb{W}_m(z'_n,w,z_m,z_n)\right]$$
$$+\sum_{y}\frac{\lambda e(y)}{N}\left[\max\left\{\widehat{W}_m(w,z_m,z_n),\min\left\{\nu_{z_m}(y),\gamma\nu_{z_m}(y)+(1-\gamma)u_m(z_m,z_n)\right\}\right\}\right.$$
$$\left.-\widehat{W}_m(w,z_m,z_n)\right]$$

Finally for a worker in a firm (z_m,z_n) that is getting wage w we have

$$\widetilde{W}_n(w,z_m,z_n)=\widehat{W}_n(w,z_m,z_n)+\delta\left[U(z_n)-\widehat{W}_n(w,z_m,z_n)\right]$$
$$+\left(\delta+\sum_{y}\frac{\lambda e(y)}{N}h_m(z_m,z_n;y)\left[\widehat{W}_n(w,0,z_n)-\widehat{W}_n(w,z_m,z_n)\right]\right)$$

$$+\sum_{z'}\frac{\lambda_u e_u(z')}{N}h_u(z';z_m,z_n)\mathbb{W}_n(z'_m,w,z_m,z_n)$$

$$+\sum_{z'_m}\frac{\lambda e(z'_m,0)}{N}h_m(z'_m;z_m,z_n)\mathbb{W}_n(z'_m,w,z_m,z_n)$$

$$+\sum_{z'_n}\frac{\lambda e(0,z'_n)}{N}h_n(z'_n;z_m,z_n)\mathbb{W}_n(z'_n,w,z_m,z_n)$$

$$+\sum_{y'_t}\frac{\lambda e(y'_t)}{N}\left[h_m(y'_t;z_m,z_n)\mathbb{W}_n(z'_m,w,z_m,z_n)+h_n(y'_t;z_m,z_n)\mathbb{W}_n(z'_n,w,z_m,z_n)\right]$$

$$+\sum_{y}\frac{\lambda e(y)}{N}\Big[\max\left\{\widehat{W}_n(w,z_m,z_n),\min\left\{\nu_{z_n}(y),\gamma\nu_{z_n}(y)+(1-\gamma)u_n(z_m,z_n)\right\}\right\}$$
$$-\widehat{W}_n(w,z_m,z_n)\Big]$$

In the Reallocation stage, after the Search and Match, a manager in a firm $(z_m, 0)$ faces the following continuation value:

$$\widehat{W}_m(w,z_m,0)=\max\Big\{U(z_m),\ \ r_m\widehat{W}_n(w,0,z_m)$$
$$+(1-r_m)\min\left\{d_mU(z_m)+(1-d_m)\widehat{W}_m(w,z_m,0),V(z_m,0)-V(0,0)\right\}\Big\}$$

Similarly for a worker alone in a firm $(0, z_n)$

$$\widehat{W}_n(w,0,z_n)=\max\Big\{U(z_n),\ \ r_n\widehat{W}_m(w,z_n,0)$$
$$+(1-r_n)\min\left\{d_nU(z_n)+(1-d_n)\widehat{W}_n(w,0,z_n),V(0,z_n)-V(0,0)\right\}\Big\}$$

As for workers with coworkers, a manager in a firm (z_m, z_n) gets:

$$\widehat{W}_m(w,z_m,z_n)=\max\{U(z_m),d_n\left(r_mW_n(w,0,z_m)+(1-r_m)W_m(w,z_m,z_n)\right)$$
$$+r_mr_nW_n(w,z_n,z_m)$$
$$+(1-d_n-r_nr_m)\min\{(d_m+d_{mn})U(z_m)+(1-d_m-d_{mn})W_m(w,z_m,z_n),$$
$$V(z_m,z_n)-V(0,z_n)\}\}$$

Finally, a worker in a firm (z_m, z_n)

$$\widehat{W}_n(w,z_m,z_n)=\max\{U(z_n),d_m\left(r_nW_m(w,z_n,0)+(1-r_n)W_n(w,0,z_n)\right)+$$
$$+r_nr_mW_m(w,z_n,z_m)$$
$$+(1-d_m-r_nr_m)\min\{(d_n+d_{mn})U(z_n)+(1-d_n-d_{mn})W_n(w,z_m,z_n),$$
$$V(z_m,z_n)-V(z_m,0)\}\}$$

To close the workers optimal values we write below the more standadt values at productions and shocks at the begginng of next period. A manager in a firm $(z_m, 0)$ with wage w gets

$$W_m(w, z_m, 0) = w + \beta(1-\sigma)\widetilde{W}_m(w, a', z_m, 0)$$

For worker alone in a firm $(0, z_n)$

$$W_n(w, 0, z_n) = w + \beta(1-\sigma)\widetilde{W}_n(w, a', 0, z_n')$$

For manager in a firm (z_m, z_n)

$$W_m(w, z_m, z_n) = w + \beta(1-\sigma)\left[\sum_{z_n'} Q(z_n'|z_n, z_m)\left(\sigma\widetilde{W}_m(w, a', z_m, 0) + (1-\sigma)\widetilde{W}_m(w, a', z_m, z_n')\right)\right]$$

For worker in a firm (z_m, z_n)

$$W_n(w, z_m, z_n) = w + \beta(1-\sigma)\left[\sum_{z_n'} Q(z_n'|z_n, z_m)\left(\sigma\widetilde{W}_n(w, a', 0, z_n) + (1-\sigma)\widetilde{W}_n(w, a', z_m, z_n')\right)\right]$$